How does the market variance risk premium vary over time? Evidence from S&P 500 variance swap investment returns[§]

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Abstract

We explore whether the market variance risk premium (VRP) can be predicted. First, we propose a novel approach to measure VRP which distinguishes the investment horizon from the variance swap's maturity. We extract VRP from actual rather than synthetic S&P 500 variance swap quotes, thus avoiding biases in VRP measurement. We find that a deterioration of the economic and trading conditions increases VRP. These relations hold both in- and out-of-sample for various maturities and investment horizons and they are economically significant. Volatility trading strategies which condition on the detected relations outperform popular buy-and-hold strategies even after transaction costs are considered.

JEL Classification: G13, G17

Keywords: Economic conditions, Predictability, Trading activity, Variance swaps, Variance risk premium, Volatility trading

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1 Introduction

The variance risk premium (VRP) is the reward required by a risk averse investor for being exposed to the risk stemming from random changes in the variance of the risky asset and from jumps in its price (Todorov, 2010, Bollerslev and Todorov, 2011). Surprisingly, there is a paucity of research on whether the market VRP is predictable. This paper investigates whether the Standard and Poor's (S&P) 500 VRP can be predicted.

Identifying the predictors of VRP's time variation enhances our understanding of the predictability of the total equity risk premium which includes the equity risk premium (arising from continuous fluctuations in the price of the risky asset) and VRP (Bollerslev et al., 2009, Chabi-Yo, 2012). Exploring whether the market VRP is predictable is also of importance to market participants who trade variance.[1] Typically, variance trading strategies yield a negative market VRP indicating that short volatility positions are profitable (e.g., Coval and Shumway, 2001, Bakshi and Kapadia, 2003, Driessen and Maenhout, 2007, Ait-Sahalia et al., 2013). However, these positions are vulnerable to sharp increases in market volatility; this was highlighted over the recent 2008 crisis where the single names variance swap market dried up (Carr and Lee, 2009, Martin, 2013). Therefore, predicting the time variation of VRP over time will help market participants to construct profitable volatility trading strategies and to avoid taking excessive risks.

We examine the predictability of the market VRP comprehensively and we make four main contributions. First, we propose a novel approach to compute VRP. We calculate VRP as the conditional expectation of the profit and loss (P&L) from a

[1] Anecdotal evidence suggests that trading volatility has become particularly popular over the last decade. This can be attributed to the development of a number of implied volatility indexes which enable the development of volatility dependent products such as volatility futures, volatility options and volatility exchange traded funds. The new products improve upon the traditional class of volatility strategies conducted via index options. The development of variance and volatility swap markets has also expanded the menu of volatility strategies even further.

long position in a T-maturity S&P 500 variance swap (VS) contract held over an investment horizon $h \leq T$.[2] The previous literature defines and measures VRP assuming that the position in a variance trade is held up to the maturity of the variance trading vehicle, i.e. $h = T$. However, in practice the position in a variance trade may be closed well before its maturity. Our method takes this stylized fact into account and thus it generalizes the conventional approach by measuring VRP for investment horizons that may be shorter than the VS maturity.

Second, we implement our proposed method and calculate VRP by using actual VS quotes written on the S&P 500. Previous studies measure VRP by employing synthetic VS rates (e.g., Bollerslev et al., 2009, 2012, Carr and Wu, 2009, Bekaert and Hoerova, 2013, Fan et al., 2013, Neumann and Skiadopoulos, 2013).[3] In line with the theoretical results of Britten-Jones and Neuberger (2000), Jiang and Tian (2005) and Carr and Wu (2009), these rates are synthesized using a particular portfolio of European options. However, the replication process of the VS rate yields a bias in the VRP calculation because of the failure to account for jumps in the underlying asset (Demeterfi et al., 1999, Ait-Sahalia et al., 2013, Bondarenko, 2013, Du and Kapadia, 2013), the finite number of traded options (Jiang and Tian, 2005, 2007) and the artificially induced jumps by the replication algorithm (Andersen et al.,

[2]A variance swap (VS) is a contract that has zero value at inception. At maturity, the long side of the VS receives the difference between the realized variance over the life of the contract and a fixed rate, called the variance swap rate, determined at the inception of the contract. A VS is a pure bet on variance and hence its market rates provide the natural venue to calculate VRP over a given investment horizon (for a review of VSs, see Demeterfi et al., 1999).

[3]Alternatively, previous studies compute VRP by taking a parametric approach where an assumed model is fitted either to market option prices (see among others, Bates, 2000, Chernov and Ghysels, 2000, Todorov, 2010, Bollerslev et al., 2011) or it is fitted to VS prices (Amengual, 2009, Egloff et al., 2010, Ait-Sahalia et al., 2012). There is also a number of studies which compute VRP by testing whether variance is priced in the cross-section of the asset returns (see e.g., Ang et al., 2006, Cremers et al., 2012). However, the computed VRP again depends on the assumed asset pricing model.

2011). Our VS data allow us to verify that this bias is significant and they enable us to circumvent it, thus providing reliable VRP estimates.[4]

Third, we explore the market VRP's predictability by taking a unified approach being navigated by financial theory and previous empirical evidence. In particular, we investigate whether VRP can be predicted by (1) the variation in the volatility of the S&P 500 returns, (2) stock market conditions, (3) economic conditions and (4) trading activity conditions. VRP is expected to be predicted by variables falling in the (1) - (4) categories from a theoretical as well as from an empirical perspective. (1) is founded on the fact that VRP stems from variance changes and hence we consider it as a stand-alone category. Variance changes either due to its negative correlation with the market (Cox, 1996) or due to its independent variation stemming from a separate source of risk (e.g., Heston, 1993). In addition, Eraker (2008), Bollerslev et al. (2009), Bekaert and Engstrom (2010) and Drechsler and Yaron (2011) models imply that stock and macroeconomic factors correlated with the volatility and the volatility of volatility of the aggregate consumption growth should also predict VRP. Furthermore, Bakshi and Madan (2006), Chabi-Yo (2012) and Feunou et al. (2013) models predict that VRP is expected to be predicted by factors nested within the (1) - (4) setting.[5]

Fourth, we complement Egloff et al. (2010) and Ait-Sahalia et al. (2013) by providing evidence on the properties of investment strategies in the index variance swap markets. The previous literature has studied the performance of volatility

[4]Interestingly, three recent papers show that VS rates can be synthesized by market option prices even in the presence of jumps. This is feasible once either the payoff of the traded VS is proxied by a correlated payoff of a specific functional form and a certain trading strategy in options and in the underlying asset is followed (Bondarenko, 2013, Martin, 2013, Mueller et al., 2013) or the trading strategy in European option prices is modified (Du and Kapadia, 2013). However, both approaches require a continuum of traded options; this condition is not met in practice.

[5]We distinguish between stock market conditions and economic conditions in line with anecdotal evidence which suggests that the state of the stock market and that of the economy may be disconnected (e.g., a booming stock market may coincide with a poor economic state).

strategies by focusing mainly on option and volatility futures markets (e.g., Coval and Shumway, 2001, Bakshi and Kapadia, 2003, Driessen and Maenhout, 2007, Konstantinidi et al., 2008).

To address our research question, first we compute the market VRP from different T-maturity VS contracts and across different investment horizons h (term structure of VRP). Then, we conduct an in-sample as well as an out-of-sample analysis of models [1] - [4] which are expected to predict VRP. The out-of-sample setting is a useful diagnostic for the in-sample specification and it is interesting for an investor who would like to use the models for market-timing. Hence, we perform the out-of-sample analysis using both a statistical as well as a VS trading strategy setting.

We find a negative market VRP across the various investment horizons. The results reveal that the VRP increases in absolute terms (i.e. it becomes more negative) when the economic and trading conditions deteriorate. This holds across investment horizons and VS contracts' maturities. Our findings confirm the financial theory predictions and they are economically significant. Variance trading strategies which use VSs and take into account the economic and trading activity conditions outperform the buy-and-hold S&P 500 strategy, the short volatility strategy commonly used by practitioners and the trading strategy based on the random walk model. These findings are robust even after transaction costs are considered.

Related Literature: Being motivated by cross-sectional asset pricing models, Carr and Wu (2009), Fan et al. (2013), Nieto et al. (2013) examine the determinants of VRP within a contemporaneous rather than a predictive setting. Amengual (2009) and Ait-Sahalia et al. (2013) examine the dynamics of a (parametrically measured) term structure of VRP. However, they do not address the question whether VRP is predictable. To the best of our knowledge, only a few papers have examined whether the market VRP can be predicted, yet there are some distinct differences between these papers and ours. Adrian and Shin (2010) document that an increase in broker dealers' funding liquidity predicts a decrease in VRP. Bekaert et al. (2013) find that a lax monetary policy also decreases VRP. However, both papers use synthetic VS rates to measure VRP. On the other hand, Bollerslev et al. (2011), Corradi et al. (2013) and Feunou et al. (2013) adopt parametric models to compute VRP and they examine its dynamics among their other purposes. The three papers find that certain macro-variables, the business conditions and the term structure of the risk-neutral variance affect VRP, respectively. Nevertheless, their measurement of VRP depends on the assumed parametric model and their analysis focuses on an in-sample setting. Finally, all the above studies but Feunou et al. (2013), focus on a 30-days investment horizon whereas investors who trade volatility use longer investment horizons, as well.

The remainder of the paper is structured as follows. Section 2 describes the data. Section 3 explains the proposed method to calculate the market VRP. Section 4 describes the theoretical foundations and the empirical evidence which justify the choice of the setting to explore the predictability of VRP. Sections 5 and 6 present the in- and out-of-sample results on the statistical and economic significance of the predictors of VRP's evolution, respectively. The last section concludes.

2 Data

2.1 Variance swap rates

We obtain daily closing quotes on over-the-counter VS rates (prices) quoted in volatility terms from a major broker dealer. The obtained VS quotes are written on the S&P 500 index and they correspond to different constant times-to-maturities (2 months, 3 months, 6 months, 1 year, and 2 years). The VS data span January 4, 1996 to February 13, 2009.

Figure 1 shows the evolution of the VS rates in volatility percentage points with time-to-maturity equal to 2, 3, 6 months, 1 and 2 years. We can see that the VS rates spike upward over periods of financial turmoil. For instance, VS rates peak in late 1998 (Russian debt and Long Term Capital Management crises), in September 2001 (World Trade Center attack), and in late 2008 (sub-prime debt crisis). Note that the shorter maturity VS contracts' spikes are more pronounced than the spikes for longer maturity contracts, and the longer maturity VS contracts are smoother than the shorter maturity ones. Moreover, most of the time, the longer maturity VS rates are higher than the shorter maturity ones. The opposite holds over periods of financial turmoil where the long maturity VS rates are generally lower than the shorter maturity VS rates. This implies that the term structure of VS rates is in contango (backwardation) in normal (crisis) periods. Table 1 reports summary statistics for the VS rates across the different maturities. We can see that the average VS rate increases as the contract's maturity increases. On the other hand, the variability of VS rates decreases as the contract's maturity increases.

2.2 Other variables

We employ data for the purposes of measuring variables expected to drive VRP. First, we obtain the daily closing prices of the S&P 500 index and the trading volume of S&P 500 futures from Bloomberg. We use these data to construct the return on the S&P 500 and the $(Volume_t/Volume_{t-1})$ ratio measured separately by the trading volume of the shortest S&P 500 futures contracts and the trading volume of all the S&P 500 futures contracts.

Second, we obtain daily data on the VIX and the SKEW index from the Chicago Board of Options Exchange (CBOE) webpage. VIX and SKEW capture the risk-neutral expectation of the realized variance and the (negative) risk-neutral skewness of the S&P 500 returns over the next 30 days, respectively. Increases in SKEW signify that the risk-neutral skewness becomes more negative.

Third, we obtain daily data from the St. Louis Federal Reserve Bank website to measure the term spread (difference between the ten-years Treasury bond rate and the one-month LIBOR rate), the credit spread (difference between the yields of the

Moody's AAA and BAA corporate bonds) and the TED spread (difference between the three-months Eurodollar rate and the three-months Treasury bill rate).

Fourth, we obtain daily data on all traded options written on the S&P 500 from the Ivy DB database of OptionMetrics to construct a number of option-based variables. In particular, we construct the growth in the open interest of all traded out-of-the-money (OTM) S&P 500 put options; we define OTM puts as these with moneyness less than 0.97. We also measure the ratio of the aggregate put volume over the aggregate call trading volume (put/call ratio) and we construct the S&P 500 at-the-money (ATM) implied volatility for a synthetic constant maturity ATM option.

3 Measuring VRP from VS investment returns

3.1 The method

The VRP over an investment horizon $h = T$ is defined as

$$VRP_{t\to t+T} = E_t^P(RV_{t\to t+T}) - E_t^Q(RV_{t\to t+T}) \tag{1}$$

where P and Q are the physical and risk-neutral probability measures, respectively, and $RV_{t\to t+T}$ is the realized variance from time t to $t+T$. The $VS_{t\to t+T}$ rate of the T-maturity VS contract inaugurated at time t is defined to be the price that makes the VS to have zero value at inception, i.e.

$$VS_{t\to t+T} = E_t^Q(RV_{t\to t+T}) \tag{2}$$

Hence, equation (1) can be re-written as

$$VRP_{t\to t+T}^T = E_t^P(RV_{t\to t+T}) - VS_{t\to t+T} = E_t^P[RV_{t\to t+T} - VS_{t\to t+T}] = E_t^P[P\&L_{t\to t+T}^T] \tag{3}$$

where $P\&L^{T}_{t\to t+T}$ denotes the T-period P&L obtained from a long position on the T-maturity VS contract held from t to $t+T$. The superscript T in the term $E^P_t[P\&L^T_{t\to t+T}]$ is used to remind that VRP is obtained from a trading strategy where the contract's maturity is an additional parameter to the investment horizon one. Equation (3) shows that VRP is defined to be the conditional expectation of the P&L of a long position in a T-maturity VS held over an investment horizon $h = T$. The contract specifications of an S&P 500 VS define the realized variance (RV) over the interval $[t,\, t+T]$ to be

$$RV_{t\to t+T} = \frac{252}{T}\sum_{i=1}^{T} ln(\frac{S_{t+i}}{S_{t+i-1}})^2 \tag{4}$$

where S_t is the closing price of S&P 500 on day t.

Inspection of equation (3) reveals that VRP is measured by assuming implicitly that the position in VS is held until its maturity; the previous literature has adopted this implicit assumption. However, in practice, the long position in a variance trade may be closed prior to its maturity, i.e. it can be held over an investment horizon $h < T$. Our proposed measure of VRP takes this stylized fact into account and it distinguishes the investment horizon from the maturity of the VS contract used to extract VRP from. In particular, we measure the market VRP based on the following proposition.

Proposition 1. *The $VRP^T_{t\to t+h}$ obtained from a long position on the T-maturity VS contract held from t to $t+h$ ($h \leq T$) is the conditional expectation of the P&L formed at time t under the P probability measure, i.e.*

$$VRP^T_{t\to t+h} = E^P_t[P\&L^T_{t\to t+h}] \tag{5}$$

where $P\&L^T_{t\to t+h}$ denotes the h-period P&L obtained from a long position on the

T-maturity VS contract held from t to $t+h$ and is calculated as

$$P\&L^{T}_{t\to t+h} = e^{-r(T-h)}N[\lambda RV_{t\to t+h} + (1-\lambda)\,VS_{t+h\to t+T} - VS_{t\to t+T}] \tag{6}$$

where N is the notional value of the VS, r the risk-free rate, $\lambda = \frac{h}{T}$ is the proportion of the investment horizon over the time-to-maturity of the traded contract, $VS_{t\to t+T}$ is the VS rate of a contract initiated at time t that matures at time $t+T$, $RV_{t\to t+h} = \frac{252}{h}\sum_{i=1}^{h} ln(\frac{S_{t+i}}{S_{t+i-1}})^2$ is the realized variance of the underlying asset's return distribution from t to $t+h$, and S_t is the closing price of the underlying asset on day t.

Proof. See Appendix A. ■

Equation (6) lies in the centre of our proposed approach. It shows that the $P\&L^{T}_{t\to t+h}$ is a weighted sum of the "accrued" realized variance from t to $t+h$ and the "capital gain" which is difference in the two T-maturity VS rates prevailing at times t and $t+h$, respectively. Interestingly, the VS $P\&L^{T}_{t\to t+h}$ is analogous to the $P\&L^{T}_{t\to t+h}$ from a long position in a T-maturity bond held over an h period ($h < T$) which equals the accrued interest and the P&L from the marked-to-market bond position over the h-period.

Notice that our proposed approach to measure VRP is more general than the conventional VRP measure. In the special case where $h = T$, the conditional expectation of the $P\&L^{T}_{t\to t+h}$ defined by equation (6) becomes the conventional definition of VRP depicted by equation (1).

3.2 Implementation

We calculate every day the $P\&L^{T}_{t\to t+h}$ realized from investing in the T-maturity S&P 500 VS under scrutiny for different horizons h (h = 1, 2 and T months) by using equation (6). To this end, we assume that the notional value of the VS contract is one and that the risk-free rate of interest is zero. The latter assumption does not affect our results and it is in line with market practice; unreported results show that the correlation between the P&L assuming r=0 and the discounted P&L is almost one (0.99). To implement equation (6), all terms but $VS_{t+h\to t+T}$ are observable because VS rates are quoted for constant times-to-maturity. Hence, we need to interpolate the $VS_{t+h\to t+T}$ rates for any maturity T and investment horizon h. In line with Carr and Wu (2009) and Egloff et al. (2010), we use the linear in the total variance interpolation method to obtain the value of $VS_{t+h\to t+T}$.[6]

Figure 2 shows the time variation of the P&Ls from investing in the VSs over h = 1 months, 2 months and T months. We can see that the evolution of the P&Ls is similar across the different investment horizons and across the different contract maturities. We can also see that the P&L spikes upwards evidently in late 2007 which corresponds to the beginning of the sub-prime 2007-2009 debt crisis and they also become positive and they jump upwards in late 2008 around the Lehman Brothers' default. This shows the risks from taking short volatility trading positions; in the event of a stock market crisis when volatility increases, the short positions in volatility can be catastrophic.

Table 2 reports the summary statistics of the VS P&L from investing in VS contracts of different maturities and over different investment horizons (h = 1, 2 and T months, panels A, B and C, respectively); the previous literature has not examined the effect of the contract maturity and the effect of the investment horizon on VRP. A number of observations can be drawn. First, the average VS P&L (i.e. the unconditional market VRP) is negative and it is statistically significant in almost all cases across h and T. The only exception occurs for the two-years VS contract for one and two months investment horizons, albeit in these cases the average P&L

[6]At time t, the total variance interpolation method amounts to obtaining the T-maturity VS rate ($VS_{t\to t+T}$) from the traded T_i and T_{i+1}-maturity VS contracts ($VS_{t\to t+T_i}$ and $VS_{t\to t+T_{i+1}}$, with $T_i < T < T_{i+1}$) as follows:

$$VS_{t\to t+T} = \frac{1}{T}\left[\frac{(T-T_i)}{(T_{i+1}-T_i)}(T_{i+1}VS_{t\to t+T_{i+1}} - T_iVS_{t\to t+T_i}) + T_iVS_{t\to t+T_i}\right]$$

is statistically insignificant. The evidence for a negative market VRP is in line with the S&P 500 negative VRP reported by the previous literature for the case of $h = T = 30$ days (e.g., Carr and Wu, 2009, Neumann and Skiadopoulos, 2013) and it indicates that on average it is profitable to sell S&P 500 VSs. In particular, for each \$100 of notional, the market VRP reaches its maximum value by shorting the 2 months maturity VS contract and holding this to its maturity (bi-monthly VRP of -\$1.1, i.e. -1.1%). Two remarks are in order at this point. First, the reported VRP's are annualized because the VS rates and the realized variances used to calculate the VS P&L are already annualized. Second, the sizes of our obtained VRPs cannot be compared to the ones obtained by the previous literature. This is because in the earlier studies, the focus had been on a 30-days maturity contracts and on a 30-days investment horizons; a 30-days maturity VS rate is not included in our data.

Second, the unconditional VRP increases as we move from the longer to the shorter maturity VS contracts for any given investment horizon h. Hence, on average it is more profitable to short shorter than longer maturity VS. Moreover, the market VRP is greater in the two than in the one-month investment horizon in absolute terms. In addition, the VS P&Ls are not normally distributed; they exhibit a positive skewness and an excess kurtosis which are higher for the shorter maturities VSs. Unreported results show that the P&Ls are positively correlated across investment horizons and maturities; the smallest correlation is 0.58. The correlation is higher between the P&Ls from investing in VS contracts with maturities that are close to each other and they share the same investment horizon.

3.3 Computing VRP: A comment on biases

At this point, a remark on the existence of biases in computing VRP is in place. The VRP computation requires the VS rate as an input which equals the $E^Q(RV)$ [equation (2)]. Given that data on actual VS rates are not available from data vendors, typically the previous literature computes VRP by synthesizing the VS rates via a trading strategy in European options and futures (for the theoretical underpinnings, see Britten-Jones and Neuberger, 2000, Jiang and Tian, 2005, Carr and Wu, 2009, and references therein); the strategy mimics the VIX construction algorithm (Jiang and Tian, 2007). However, this may yield a bias in the calculation of VRP for at least three reasons.

First, the synthesized VS rate is a biased estimator of $E^Q(RV)$ in the presence of jumps in the underlying S&P 500 index (Demeterfi et al., 1999, Ait-Sahalia et al., 2013, Bondarenko, 2013, Du and Kapadia, 2013). More specifically, synthesized VS rates underestimate actual VS rates when downward jumps dominate with the bias being proportional to the jump intensity (Du and Kapadia, 2013). Second, there are numerical errors in synthesizing the VS rates (Jiang and Tian, 2007). Finally, Andersen et al. (2011) document that the VIX algorithm creates artificially jumps and it is particularly unreliable during periods of market stress when it's informational content as a gauge of the investor's fear is needed most. The authors conclude that "*the quality of the risk premium measures [based on VIX] are similarly degraded*". Our computed VRP bypasses the above constraints because we implement equation (6) by using actual VS quotes and hence we do not need to synthesize the VS rates.

To demonstrate that the $P\&L^T_{t\to t+h}$ constructed from the actual VS rates differs from the $P\&L^T_{t\to t+h}$ constructed from the synthetic ones, we synthesize the 60 and 90-days to maturity VS rates by following the Carr and Wu (2009) approach. In sum, the approach replicates the VS rate by following four steps. First, we collect the prices of OTM S&P 500 European calls and puts with maturities surrounding any targeted constant maturity. Then, for each maturity, we perform a cubic spline interpolation across the obtained option prices as a function of the strike price to obtain a continuum of option prices. Next, we calculate the integral of a certain portfolio of the collected options which yields the price of this portfolio; the portfolio price is the VS rate of the respective maturity. Finally, we derive the targeted constant maturity VS rate by interpolating linearly across the VS rates of the surrounding maturities.

Figure 3 shows the difference between the two-month maturity actual and the synthesized VS rates in volatility percentage points (panel A) and the difference between the respective $P\&L^{T}_{t\to t+h}$ constructed from the actual and synthesized VS rates (panel B) for $h = 1, 2$. Panel A shows that the two-months maturity quoted and synthesized VS rates do differ and the difference tends to be positive over time; on average this difference is 1.3 volatility points. A t-test suggests that the null hypothesis of a zero mean difference is rejected at a 1% level of significance (t-statistic = 51.7). Similarly, panel B shows that the P&Ls based on quoted and synthesized VS rates differ across all investment horizons. The unreported mean P&L difference is negative over time and increases with the investment horizon (mean difference is -0.01% and -0.56% for the one and two months investment horizon, respectively). This suggests that on average, the P&L based on the synthesized VS rates overestimates the P&L based on actual quotes. A t-test also suggests that the average P&L difference is significant only at the two months investment horizon (t-statistic = -0.16 and -32.28 for the one and two months investment horizon, respectively). Analogous findings are documented for the cases where the P&Ls are extracted from other maturity contracts.

These results corroborate Andersen's et al. (2011) conclusions and indicate that the VRP computed from synthesized VS rates suffer from biases. Hence, they should not be used for the purposes of our analysis.

3.4 Does VRP vary over time?

In this section, we test whether VRP is time-varying or constant. This is a prerequisite stage before embarking on the VRP predictability exercise. To this end, we run the following regression:

$$\lambda RV_{t\to t+h} = a + b\{-(1-\lambda)VS_{t+h\to t+T} + VS_{t\to t+T}\} + e_{t+h} \tag{7}$$

In the case where we accept the null hypothesis $H_0 : b = 1$, then in light of equation (6) this would imply that the average h-time horizon $P\&L^T_{t\to t+h}$ is constant. In the particular case where $h = T$, the regression described by equation (7) becomes

$$RV_{t\to t+T} = a + bVS_{t\to t+T} + e_{t+T} \tag{8}$$

Equation (8) is the standard expectation hypothesis regression used to check the constant risk premium hypothesis (for an application to the case of VRP measured assuming that the VS contract is held to its maturity, see Carr and Wu, 2009, Ait-Sahalia et al., 2013). Table 3 reports the estimated coefficients and t-statistics obtained from the regression (7) for the various forecasting horizons and the maturity contracts. We can see that the null hypothesis that the slope coefficient equals one ($H_0 : b = 1$) is rejected in all cases. This suggests that the $P\&L^T_{t\to t+h}$ varies over time and it confirms the similar evidence provided by Carr and Wu (2009) and Ait-Sahalia et al. (2013).

4 Predictability of VRP: Theoretical background

In the previous section we found that VRP varies over time. Next, we relate the VRP's time variation to a number of variables founded on theoretical and empirical considerations, i.e.

$$VRP^T_{t\to t+h} = E^P_t[P\&L^T_{t\to t+h}] = c^T_0 + c^T_1 X_t \tag{9}$$

where X is a $(n \times 1)$ vector of the VRP drivers, c^T_0 is a scalar constant, c^T is a $(1 \times n)$ vector of constant coefficients and the superscript T reminds that VRP is extracted from the T-maturity VS. Equation (9) shows that we examine the VRP time variation in a predictive setting because we use the information known up to time t to explain the VRP movements over the time interval $[t,\, t + h]$.

The variables contained in vector X are related to the variation of the S&P 500 volatility, the stock market conditions, the state of the economy and the trading activity. A number of models predict that VRP should be driven by factors related to these four conditions (e.g., Bakshi and Madan, 2006, Eraker, 2008, Bollerslev et al., 2009, Bekaert and Engstrom, 2010, Drechsler and Yaron, 2011, Chabi-Yo, 2012, and Feunou et al., 2013).

Table 4 provides a list of the drivers of VRP and the way they affect it. In Sections 4.1 - 4.4 we outline briefly the rationale underlying the use of these variables as VRP predictors and how we measure them. Notice that for the purposes of our discussion, we fix the terminology hereafter as follows. Given that the market VRP is on average negative, we follow the VRP literature and define an increase in VRP to signify that the negative VRP becomes more negative.

4.1 Variation in the volatility of the S&P 500 returns

We consider the correlation ($Corr$) of variance changes with the S&P 500 returns and the variance of volatility (VoV) of the S&P 500 returns as the natural drivers of VRP's time-variation and thus, we include them in a stand-alone category.

VRP is generated by random changes of the underlying asset's variance. These random changes stem from two sources. First, the variance may vary stochastically due to its negative correlation with the market (proxied by $Corr$ in our setting). This arises for instance within the constant elasticity of variance model (Cox, 1996) where the variance is correlated with the stock price and it is driven by the same shocks as returns. Second, the variance may vary stochastically due to a separate source of risk (e.g., Heston, 1993, proxied by VoV in our setting). Eraker (2008), Bollerslev et al. (2009) and Drechsler and Yaron (2011) also assume that an independent factor drives the stochastic evolution of the variance.

$Corr$ has been documented to be negative (leverage effect). We define an increase in $Corr$ to signify that the negative $Corr$ becomes more negative; this is in analogy to the convention we use for VRP. We expect VRP to be positively correlated with $Corr$, i.e. we expect VRP to increase (i.e. become more negative) when $Corr$ also increases (i.e. becomes more negative). This is because an investor who holds a stock position pays a negative VRP as an insurance premium because the decline in the stock return can be hedged by a long position in a VS which benefits from the rise in volatility. Hence, the negative VRP she wills to pay becomes more negative the greater the negative $Corr$ becomes because this increases the hedging effectiveness of the VS. We measure $Corr$ as the rolling correlation of the daily S&P returns and the VIX changes over the past year.

Regarding VoV, we expect VRP to increase in magnitude (i.e. to become more negative) as VoV increases. In other words, we expect to find a negative correlation between the negative average VRP and VoV. This is because the greater the variation of the variance, the greater the insurance risk premium the investor is prepared to pay. We construct VoV as the difference between the VS rate measured in variance terms and the squared volatility swap rate ($VolS$) for a time-to-maturity equal to two months. This is because under the Q-probability measure:

$$VoV_t = var_t^Q(\sigma) = E_t^Q(\sigma^2) - E_t^Q(\sigma)^2 = VS_t - VolS_t^2 \tag{10}$$

Carr and Lee (2009) show that $VolS$ is well approximated by the at-the-money (ATM) implied volatility. Hence, we measure $VolS$ by the S&P 500 ATM implied volatility.

4.2 Stock market conditions

Next, we consider VIX, the S&P 500 return, the S&P 500 risk-neutral skewness, and the S&P 500 ex-ante VRP as stock market variables expected to predict the VRP time variation.

We expect a negative relation between VRP and stock market volatility; an increase in VIX will increase VRP in magnitude, i.e. it will make it more negative. Eraker (2008) and Chabi-Yo (2012) confirm this prediction by developing general and partial equilibrium models, respectively, where VRP is derived as a function of the market volatility. In addition, a number of papers (e.g., Heston, 1993, Egloff et al. 2010) assume that the magnitude of VRP increases as the volatility increases. Therefore, testing the relation between VIX and the VRP provides a test of this assumption and Chabi-Yo's (2012) theoretical implications.

We expect a positive relation between the S&P 500 return and the magnitude of VRP. This is because a decrease in the stock return will increase volatility due to the leverage effect. This will in turn increase the magnitude of VRP (i.e. it will make it more negative) given the expected positive relation between the magnitude of VRP and volatility. We measure S&P 500 returns over the past h-months to match the horizon of the stock return with the investment horizon.

We also consider the risk-neutral skewness of S&P 500 return distribution as a VRP predictor. We expect VRP to become more negative when the risk-neutral skewness becomes more negative. This is because a negative risk-neutral skewness captures the market participants fears for downward jumps in asset prices (Bakshi and Kapadia, 2003). In the occurrence of such a rare event, volatility will increase and the buyer of a VS will benefit. Hence, the buyer of the VS is willing to pay a greater VRP to take advantage of these downward jumps in S&P 500; Todorov (2010), Bollerslev and Todorov (2011), and Ait-Sahalia et al. (2013) also find that VRP reflects jump fears. We use the CBOE skew index (SKEW) to measure the risk-neutral skewness of the S&P 500 return distribution. According to the construction methodology of the SKEW, increases in its value signify that the risk-neutral skewness becomes more negative. Consequently, the relation between VRP and the CBOE SKEW index is expected to be negative.

Finally, we consider the ex-ante VRP as a predictor of the (ex-post) VRP. This is because a number of studies (e.g., Bollerslev et al., 2009, 2012, Mueller et al., 2011) find that the ex-ante VRP predicts the returns of various asset classes. Hence, our approach can also be viewed as a study about whether VRP forecasts the returns of an additional asset class like variance swaps. Assuming that the ex-ante VRP contains information about the ex-post VRP, we expect to find a positive relation between them. In the particular case where the ex-ante VRP is an unbiased and efficient forecast of the ex-post VRP, then the constant and the slope coefficient in the regression of the ex-post VRP on the ex-ante VRP are zero and one, respectively.

The ex-ante VRP is defined in equation (1). We measure the ex-ante VRP over the next h-months to match the horizon of the ex-ante VRP with the investment horizon (i.e. the horizon of the ex-post VRP). We use a GARCH(1,1) model to construct the expected realized variance under the P-probability measure (see Appendix B). For the risk-neutral expectation of the realized variance, we use the squared VIX index for $h = 1$ month and the squared VS rates for the remaining investment horizons. This is because the square of VIX equals the risk-neutral expectation of the realized variance (Jiang and Tian, 2007) and the actual VS rates are quoted in volatility terms.

4.3 Economic conditions

Regarding the economic conditions, we consider the slope of the yield curve and the credit spread as variables which affect VRP's dynamics. This is because VRP is counter-cyclical (Bollerslev et al., 2011, Bekaert et al., 2013, Corradi et al., 2013). The slope of the yield curve and the credit spreads have been found to predict the state of the economy (see Estrella and Hardouvelis, 1991, and Gomes and Schmidt, 2010, respectively). In fact, Bollerslev et al. (2011) find that the credit spread drives the VRP dynamics. As the term structure flattens and/or the credit spread increases, VRP is expected to increase in magnitude, i.e. to become more negative;

a flatter term structure and a higher credit spread predict a recession.[7]

4.4 Trading activity

We investigate the predictive ability of five trading daily activity variables: (a) the trading volume of the shortest S&P 500 futures contract, (b) the trading volume of all S&P 500 futures contracts, (c) the open interest of the traded out-of-the-money (OTM) S&P 500 put options, (d) the ratio of the trading volume of all traded S&P 500 puts to that of all traded calls maturities (put/call ratio) and (e) the TED spread.

The daily trading volume of the shortest S&P 500 futures contract proxies the underlying assets's volume. We consider this as a predictor of VRP's dynamics because the volume of the underlying stock index has been found to forecast positively the negative skewness of the physical return distribution (Chen et al., 2001). The latter is contemporaneously positively related to VRP (Bakshi and Madan, 2006). Therefore, we expect a negative correlation between changes in the underlying asset's trading volume and VRP; an increase in the trading volume will increase VRP, i.e. it will make it more negative.

We examine whether the trading volume of all S&P 500 futures contracts predicts the $P\&L^T_{t\to t+h}$ time variation. We expect the magnitude of VRP to decrease, i.e. VRP to become less negative, as the aggregate S&P 500 futures trading volume increases. This is because the latter implies lower volatility for the S&P 500 (Bessembinder and Seguin, 1992). Hence, the smaller volatility is, the smaller VRP will be (see Section 4.2).

The daily open interest of OTM S&P 500 put options proxies the investors'

[7]Interestingly, Fan et al. (2013) attach an alternative interpretation to the credit spread which yields an effect on VRP to the same direction as the state of the economy interpretation: an increase in the credit spread indicates that market makers are less willing to take on additional risk and as a result this will be manifested by an increase in VRP, too. This allows taking into account the role of financial intermediaries for the purposes of explaining the VRP dynamics.

demand for hedging downside tail risk. The greater our open interest variable is, the greater VRP is expected to be in order for the long hedgers to entice investors to share their risk.

We consider the put/call ratio as a predictor because it is regarded as a measure of the market sentiment which has been found to affect risk-neutral skewness (Han, 2008). In the case where the market is pessimistic, the volume of puts is expected to be greater than the volume of calls. This is because investors expect the market to decline and thus the risk-neutral probability density function will appear to be negatively skewed. Consequently, we expect increases in the put/call ratio to make VRP more negative, i.e. to increase it in magnitude since there is a negative relation between VRP and risk-neutral skewness as explained in Section 4.2.

Finally, we investigate the TED spread as a predictor of VRP. The TED spread measures traders' funding liquidity. The greater TED spread is, the greater funding illiquidity is and hence the harder is for an investor to keep funding her activities and stay in the market; under this perspective, the TED spread is related to the trading activity. We expect VRP to increase in magnitude as the TED spread increases. This is because broker dealers are short in index options (Gârleanu et al., 2009) and they receive VRP as a compensation to hold these in their inventories. In the case where broker dealers face funding liquidity constraints, it is harder for them to take a short position in a VS and hence long hedgers need to offer them a greater VRP to entice them to do so. Adrian and Shin (2010) confirm this prediction by finding that broker dealers' funding liquidity predicts VRP.

5 Predicting VRP: In-sample evidence

5.1 Single predictor models

To investigate whether the factors discussed in Section 4 drive VRP's time variation, first we consider the following single predictor regression for each T-maturity VS contract and for each investment horizon h:

$$P\&L^T_{t\to t+h} = c^T_0 + c^T_i X_{it} + \epsilon^T_{it+h} \tag{11}$$

where X_{it} denotes the i-th predictor variable, and c^T_0 and c^T_i are constants. The conditional expectation of the left-hand-side of equation (11) delivers the conditional $VRP^T_{t\to t+h}$ defined by equation (5) as a function of X_{it}. In line with Goyal and Welch (2008), first we run single predictor models and then we rely on multiple predictors models. The single predictor setting allows revealing the marginal effect of the individual predictor variables. We consider VS contracts with different maturities (T=2, 3, 6, 12 and 24 months), and alternative investment horizons (h=1, 2 and T months).

We estimate equation (11) by using daily observations of the realized $P\&L^T_{t\to t+h}$ and X_{it}. We measure X_{it} from January 4, 1996 to December 31, 1999 in all cases, i.e. over a common period across all maturity VS contracts and all investment horizons. This corresponds to a sample period for the P&Ls that differs for each investment horizon since P&Ls are observed on day $t + h$ (and not t). Therefore, results are not comparable across investment horizons. The rest of the data will be used for the models out-of-sample evaluation to be conducted subsequently in Section 6.

Table 5 reports the estimated coefficients of equation (11) and Newey-West t-statistics for any given considered variable and investment horizon across the various VS maturities; Panels A, B, C and D correspond to the volatility variation, stock market, economic and trading activity conditions, respectively.[8] We can see that all

[8] An alternative way of conducting statistical inference in the presence of overlapping observations would be to employ Hodrick's (1992) standard errors. However, this is not possible in our case because of the nature of the dependent variable. This is because Hodrick's (1992) standard errors are based on the assumption that the regressand variable is measured over h periods and can be decomposed into the sum of single period variables (see Hodrick, 1992, pages 361 - 362). This does not hold in our case though because $P\&L_{t\to t+h}$ is not equal to

volatility variation, economic conditions and stock market condition variables affect the VRP time variation when they are considered as predictors in a stand-alone fashion. In the case of the trading activity variables, only the aggregate S&P 500 futures volume and the TED spread are significant. The R^2 is high in most cases. In particular, it takes the greatest values in the case of VoV and VIX (maximum R^2 is 76% and 68%, respectively). On the other hand, it takes the lowest values for the trading activity variables where it is close to zero in all but one (i.e. TED) cases. The high R^2 for VRP is in contrast to the one obtained by studies on the predictors of the equity risk premium (e.g., Goyal and Welch, 2008, find an R^2 lower than 10%).

The estimated coefficients of the significant VRP drivers have the same sign across maturity contracts for any given investment horizon. Furthermore, the estimated coefficients have the expected sign discussed in Section 4 across all VS maturities and investment horizons. In particular, an increase in $Corr$ (i.e. $Corr$ becomes less negative) reduces VRP in magnitude (i.e. VRP becomes less negative) whereas an increase in VoV decreases VRP (i.e. VRP becomes more negative). Similarly, a deterioration in the stock market conditions (i.e. a decrease in the S&P 500 return, an increase in VIX and an increase in CBOE SKEW) increases VRP in magnitude. We also find that VRP is countercyclical. In addition, an increase in the ex-ante VRP increases the ex-post VRP in magnitude. Finally, a decline of trading activity (i.e. a decrease in the aggregate S&P 500 futures volume and an increase in the TED spread) increases VRP in magnitude.

In sum, our findings confirm the predictions of financial theory. They show that the negative VRP is predicted to become more negative as VoV increases, the stock market and economic conditions deteriorate, and the trading activity decreases. The results from our VRP measures extend the findings of Corradi et al. (2013) where their parametrically measured VRP is found to be countercyclical with the state

$P\&L_{t\to t+1} + P\&L_{t+1\to t+2} + ... + P\&L_{t+h-1\to t+h}$.

of the economy. Our findings also extend the literature on the use of VRP as a predictor for the future returns of a number of asset classes (e.g., Bollerslev et al., 2009, 2012, Mueller et al., 2011, Bekaert and Hoerova, 2013) because we find that the ex-ante VRP forecasts the returns of variance swaps, too. Notice that our results document that a reverse relation also holds, i.e. it is not just that VRP predicts stock market returns but the stock market conditions also predict VRP.

5.2 Single predictor models: Robustness tests

We assess the robustness of the results documented in the previous section by considering alternative measures for the volatility variation and stock market conditions measures. In the case of the volatility variation model, we measure $Corr$ as the rolling correlation of the daily S&P returns and VIX changes over one month as opposed to one year. We also examine various VoV measures. First, we construct VoV by using equation (10) for different horizons (i.e. one month and three months as opposed to two months used previously); in the case of the one-month horizon, we use VIX squared to proxy the VS rate. Second, we follow Baltussen et al. (2013) and define VoV alternatively as follows:

$$VoV_t^{Baltussen\ et\ al.(2013)} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\sigma_i - \overline{\sigma}_t)^2}}{\overline{\sigma}_t} \tag{12}$$

where σ is a measure of stock return volatility, $\overline{\sigma}_t$ is the average volatility over the past month and $n = 21$ is the number of volatility observations over the past month. To construct this measure we consider alternative volatility measures: VIX , ATM implied volatility and GARCH(1,1) volatility forecasts. In the case of the ATM implied volatility and the GARCH(1,1) forecasts we examine various horizons; one, two and three months for the former and up to one year for the latter (i.e. 1 month, 2 months, 3 months, 6 months and 1 year).

In the case of stock market condition variables, we examine alternative stock market volatility, risk-neutral skewness and ex-ante VRP measures. First, we proxy stock market volatility with the ATM implied volatility (horizons of one, two and three months) and GARCH(1,1) volatility (horizons of 1 month, 2 months, 3 months, 6 months and 1 year). Second, we measure the risk-neutral skewness extracted from S&P 500 option prices using the Bakshi et al. (2003) model-free methodology (one and two months horizon, see Appendix C for the construction methodology) as an alternative to the CBOE SKEW variable. Note that as risk-neutral skewness increases (i.e. it becomes less negative) VRP is expected to decrease (i.e. it becomes less negative). The opposite is true for the CBOE SKEW whose construction methodology dictates that increases in its value signify that the risk-neutral skewness decreases (i.e. it becomes more negative). Third, we use alternative ex-ante VRP measures where the expected realized variance under the P- probability measure is constructed using different models (see equation(1)). To this end, we use the random walk (RW), an autoregressive of order one (AR(1)) model, the exponentially moving average (EWMA) model and the ATM implied volatility. In the case of the EWMA model, we choose the decay factor by setting the half life equal to the forecasting horizon h (see Appendix D).

Unreported results show that the previously reported single predictor in-sample findings are robust to various alternative volatility variation and stock market condition variables. In particular, $Corr$ and all VoV measures affect VRP; VRP decreases in magnitude with increases in $Corr$ and VoV.

5.3 Multiple predictor models

In the previous section we found that certain variables predict VRP when these are considered separately in a stand-alone fashion [equation (11)]. Next, we consider the effect of the statistically significant variables jointly within an in-sample setting for each one of the four types of conditions under scrutiny. The multiple predictors setting will allow detecting whether the informational content of certain drivers of

VRP is subsumed by that of other drivers. We run four multiple predictors versions of the regression described by equation (11) for each one of the type of conditions we consider:

Volatility variation model:

$$P\&L^T_{t\to t+h} = c^T_0 + c^T_1 Corr_t + c^T_2 VoV_t + \epsilon^T_{t+h} \tag{13}$$

where $Corr_t$ is the correlation between S&P 500 daily returns and daily changes in VIX over the past year and VoV_t is the variance of variance on day t. Note that $Corr_t$ and VoV_t are only moderately correlated (ρ = -0.47) so there are no concerns for multicollinearity.

Stock market conditions model:

$$P\&L^T_{t\to t+h} = c^T_0 + c^T_1 VIX_t + c^T_2 R_{t-h\to t} + c^T_3 SKEW_t + c^T_4 VRP_{t\to t+h} + \epsilon^T_{t+h} \tag{14}$$

where VIX_t is the CBOE VIX index measured at time t, $R_{t-h\to t}$ is the S&P 500 return between $t-h$ and t, $SKEW_t$ is the CBOE skewness index at time t and $VRP_{t\to t+h}$ is the ex-ante VRP between t and $t+h$. The pairwise correlations between the predictor variables are moderate and they range between -0.57 and 0.45; therefore, the specification is not subject to multicollinearity problems.

Economic conditions model:

$$P\&L^T_{t\to t+h} = c^T_0 + c^T_1 TS_t + c^T_2 CS_t + \epsilon^T_{t+h} \tag{15}$$

where TS_t and CS_t denote the term and credit spread at time t, respectively. Note that the correlation between TS and CS over the in-sample period is relatively low and negative (-0.18) and hence, these two variables may capture different aspects of the time-variation in the $P\&L^T_{t\to t+h}$. The correlation between the predictors is moderate (-0.22).

Trading activity model:

$$P\&L^{T}_{t\rightarrow t+h} = c^{T}_{0} + c^{T}_{1} TVol_{t} + c^{T}_{2} TED_{t} + \epsilon^{T}_{t+h} \tag{16}$$

where $TVOl_t$ is the growth of the aggregate S&P 500 futures trading volume and TED_t is the TED spread at time t. Note that the correlation between $TVol$ and TED is close to zero (-0.01). The correlation between the predictors is low (-0.01).

Table 6 reports the estimated coefficients of equations (13), (14), (15) and (16) for the various investment horizons across the various VS maturities. (panels A, B, C and D, respectively). Regarding the volatility variation model, we can see that VoV accounts for the time-variation in VRP in almost all cases whereas $Corr$ does not. In particular, VoV affects all VRPs for maturity contracts for investment horizons greater than one month. It has a negative effect on P&L which suggests that as VoV increases, VRP becomes more negative (i.e. it increases in magnitude). This is consistent with our findings in expected from a theoretical point of view. These findings suggest that VRP arises because there is an independent factor that drives the stochastic evolution of the variance of the S&P 500 returns. This extends the findings of Carr and Wu (2009) who document in a *contemporaneous* setting that the majority of the market VRP is generated by an independent variance risk factor.

Regarding the stock market conditions model, we can see that VIX and the S&P 500 return affect VRP whereas the other variables do not. In particular, VIX is significant in all but two cases; the only exceptions occur for the P&L obtained from holding a VS1Y and a VS2Y for one month. The S&P 500 return is significant for investment horizons greater than one month across all VS maturities. Once again, all the estimated coefficients have the expected signs. Interestingly, VIX and R subsume the significance of the SKEW and the ex-ante VRP which was documented in the previous stand-alone regressions.

Regarding the economic conditions model, we can see that both CS and TS account for the time-variation in the $P\&L^{T}_{t\to t+h}$. More specifically, CS is significant across all contract maturities and investment horizons whereas TS affects the $P\&L^{T}_{t\to t+h}$ across all VS maturities only in the case where we hold the VS contract up to its expiration. The results show that VRP is countercyclical and they confirm Corradi et al. (2013) results. Interestingly, the negative and statistically significant sign of CS can also be interpreted within a financial intermediaries setting (for a similar explanation, see also Fan et al., 2013). An increase in CS signifies that the financial intermediaries are not willing to take on excessive risk and as a result VRP has to increase in magnitude (i.e. to become more negative) so that to entice them to take on this risk. This can be the case over crisis periods where the broker dealers deleverage their balance sheets by selling risky corporate debt; this presses the bond prices down and as a result the corporate yield and hence CS increases.

Finally, regarding the trading activity model, we can see that both $TVol$ and TED affect the time-variation of VRP; the TED spread has a significant effect across all investment horizons whereas $TVol$ is significant only for investment horizons greater than two months. VRP increases in magnitude (i.e. it becomes more negative) when $TVol$ decreases. This is in line with the theories that predict that futures markets decrease spot volatility and they enhance the liquidity and depth of the spot markets (see Bessembinder and Seguin, 1992, and references therein). Finally, VRP increases in magnitude when the TED spread increases. This is consistent with a funding liquidity explanation where increases in the TED spread signify increases in the market liquidity risk and it corroborates the Adrian and Shin (2010) findings who report that the broker dealers' repos positions predict VRP. As a result, VRP also has to increase to compensate the suppliers of the VS for bearing the additional risk.

6 Predicting VRP: Out-of-sample analysis

In Section 5 we found that a number of variables and model specifications (volatility variation, stock market, economic and trading activity conditions) predict VRP within an in-sample setting. In this section we investigate whether these relations also hold in an out-of-sample setting.

We construct out-of-sample h-period P&L forecasts based on each one of the multiple predictors models (11) described in Section 5.3. We estimate each model at each point in time for any given T-maturity by using a rolling window of four years (i.e. 1,009 daily observations) and we generate the out-of-sample P&L forecast. At each time step, all predictor variables across models are measured over a common in-sample period across the various maturities and investment horizons P&Ls. The first in-sample dataset corresponds to predictors observed from January 4, 1996 to December 31, 1999. The last in-sample dataset corresponds to predictors observed from December 13, 2004 December 12, 2008.[9]

6.1 Out of sample evaluation: Statistical evaluation

We evaluate the out-of-sample forecasting performance of equation (11) for each model by using two measures: the out-of-sample R^2 (Campbell and Thompson, 2008) and the mean correct prediction (MCP) criterion. The out-of-sample R^2

[9]Alternatively, following Rapach et al. (2010), we also construct combination forecasts for any given multiple predictors model. In particular, we combine the VS P&L forecasts delivered by the single predictor models nested within any given multiple predictors model. We consider two alternative combination forecasts, namely equally and unequally weighted combination forecasts. In the case of the unequally weighted combination forecasts, we choose the weights that minimize the mean squared forecast error (see Granger and Ramanathan, 1984). This is done by regressing the realized P&L on the forecasted P&L obtained from the respective single predictor models; the estimated coefficients are used to form the combination forecasts. The results obtained from the standard multiple predictor model specifications outperform those obtained from the combination forecasts and hence, they are not reported in the interest of brevity.

shows whether the variance explained by the i-th model is greater or smaller than the variance explained by a benchmark model. We choose the random walk (RW) to be the benchmark model defined as

$$P\&L_{t\to t+h}^{T} = c_0^T + c_1^T P\&L_{t-h\to t}^{T} + \epsilon_{t+h}^{T} \tag{17}$$

Then, the out-of-sample R^2 is defined as:

$$R_i^2 = 1 - \frac{var\left(E_t^i\left[P\&L_{t\to t+h}^{T}\right] - P\&L_{t\to t+h}^{T}\right)}{var\left(E_t^{RW}\left[P\&L_{t\to t+h}^{T}\right] - P\&L_{t\to t+h}^{T}\right)} \tag{18}$$

where $E_t^i\left[P\&L_{t\to t+h}^{T}\right]$ is the forecasted $P\&L_{t\to t+h}^{T}$ obtained from the i-th model (i = 1 for the volatility variation model, 2 for the stock market conditions model, 3 for the economic conditions model, and 4 for the trading activity variables model) and $E_t^{RW}\left[P\&L_{t\to t+h}^{T}\right] = P\&L_{t-h\to t}^{T}$ is the forecasted $P\&L_{t\to t+h}^{T}$ obtained from the RW model. Positive (negative) values of the out-of-sample R_i^2 indicate that the i-th model outperforms (underperforms) the RW model. On the other hand, MCP shows the percentage of cases where a given model predicts correctly the sign of the $P\&L_{t\to t+h}^{T}$. It is the natural statistical measure for an investor who needs to decide on which model to rely to decide whether she will short or buy a VS contract. To evaluate the statistical significance of the obtained MCP figures, we use the ratio test to assess whether any model under consideration outperforms the random walk (RW) model. Notice that the random walk model is not nested within the alternative models. Therefore, typical statistical tests comparing the predictive accuracy of nested models cannot be used.

Table 7 reports the out of sample R^2 (panel A) and MCP (panel B) for any given model specification across the various VS maturities and investment horizons. Under the out of sample R^2 metric, all but the stock market conditions model outperform the RW in the vast majority of investment horizons and contract maturities. In addition, all models do well under the MCP metric with the economic conditions model delivering the best performance. In particular, the economic activity model yields the greatest MCP in most cases; MCP ranges from 54.9% to 76.6%. The only exception occurs for the one year- or a two years-maturity contract where the trading activity model performs best across all investment horizons.

6.2 Out of sample evaluation: Trading strategy

We investigate whether the evidence of statistical predictability is economically significant by considering trading strategies based on the P&L forecasts constructed from the multiple predictors models examined in Section 5 and a particular filter value F_t^T to avoid trading on noisy signals (for a similar approach, see also Gonçalves and Guidolin, 2006, Ait-Sahalia et al., 2013). To fix ideas, on day t we construct a forecast for the $P\&L_{t\to t+h}^T$ based on any given model specification. If the forecasted $P\&L_{t\to t+h}^T$ is greater (less) than a filter F_t^T $(-F_t^T)$, then we go long (short) the VS contract and we keep this position up to $t+h$. On the other hand, if the forecasted $P\&L_{t\to t+h}^T$ lies between F_t^T and $-F_t^T$, we stay out of the market. We implement this strategy over the out-of-sample period and we use a time varying filter which equals the standard deviation of the P&Ls used for the in-sample estimation for any given model specification at each time step.

To evaluate the economic significance of a given trading strategy, we calculate the Sharpe ratio (SR) by taking transaction costs into account. To this end, we use each strategy's excess returns $R_{t\to t+h}^T$ after transaction costs defined as follows:

$$R_{t\to t+h}^T = \frac{P\&L_{t\to t+h}^T \text{ after } TC}{VS_{t\to t+T} + TC} \tag{19}$$

where TC is the transaction cost in variance points. P&L correspond to excess returns assuming that the notional value of the VS contract is fully collateralised; this is a typical assumption in the literature on the computation of futures returns. Note also that in the case where we keep our position over a horizon $h < T$, we incur the transaction cost twice, whereas when we hold our position to maturity ($h = T$) we incur the transaction cost only once, i.e.:

$$P\&L^T_{t\to t+h} \text{ after } TC = \begin{cases} Position_t \times P\&L^T_{t\to t+h} - 2TC & \text{when } h < T \\ Position_t \times P\&L^T_{t\to t+h} - TC & \text{when } h = T \end{cases} \tag{20}$$

where $Position_t$ equals 1 (-1) when we enter a long (short) position in a T-maturity VS contract on day t and $P\&L^T_{t\to t+h}$ is the realized P&L after transaction costs of a position opened at t and held up to $t+h$.x We set the VS transaction costs to 0.5 volatility points (i.e. 0.25 variance points) which is the typical VS bid-ask spread (e.g., Egloff et al., 2010, and also confirmed after discussions with practitioners).

We compare the SR of any given strategy to the SR of three alternative strategies that we consider as benchmark strategies. First, following Ait-Sahalia et al. (2013) we consider a buy-and-hold strategy in the S&P 500 over various horizons h. Second, we consider a naive short volatility strategy where on day t, the investor opens a short position on a T-maturity VS contract (T = 2 months, 3 months, 6 months, 1 year, 2 years) and she keeps this position up to $t + h$ (h = 1 month, 2 months, T months). This is a popular strategy because it is well documented that the average market VRP is negative and hence, shorting variance swaps is profitable on average. Finally, we consider our trading strategy based on $P\&L$ forecasts obtained from the random walk model.

Table 8 reports the SR after transaction costs obtained for any given model specification across the various maturities and investment horizons h = 1, 2 and T (panels, A, B and C, respectively). We can see that the economic activity (trading activity) model outperforms the buy-and-hold S&P 500 strategy for a one-month (two- and T-months) investment horizon. More specifically, for h = 1 month, the volatility variation, the economic conditions and the trading activity models outperform the buy-and-hold strategy; the economic conditions model yields the greatest

SR (between 0.27 and 0.43). For $h = 2$ months, the trading activity model outperforms the buy-and-hold strategy for intermediate (i.e. 6 months)and long (i.e. 1 and 2 years) maturity VS contracts. Similar results are obtained for $h = T$ where the trading activity model outperforms the benchmark strategy for long (i.e. 1 and 2 years) maturity contracts. Notice that the stock market conditions model performs poorly across all investment horizons as it yields negative SRs across all maturity VS contracts.

Similar findings are documented in the case where we compare the SR of any strategy to the alternative benchmark trading strategy where we go short in a VS contract, as well as the trading strategy based on the RW model. For the one month investment horizon, the economic activity model outperforms both the short VS strategy and the strategy based on the RW model. For the two (T) months investment horizon, the trading activity model outperforms the short VS strategy and the RW strategy for intermediate and long (only long) maturity VS contracts.

Finally, we conduct two additional tests to check the robustness of our trading strategy results. First, we increase the VS transaction costs from 0.5 to 5 volatility points (i.e. 25 variance points). Second, instead of making an investment decision at every t, an alternative rule would be to make an investment decision every h months. This would entail taking a position at time t based on the forecasted $P\&L^T_{t\to t+h}$ and the filter value F^T_t, closing this position at $t + h$ and opening a new one the next day, i.e. on day $t + h + 1$. This trading strategy yields a scarce trading within our sample period and as a result the SR cannot be computed. Nonetheless, in the case where this strategy is applied with $F^T_t = 0$ for every t (i.e. no filter), the number of trades increases for $h = 1$ and 2 months. In either robustness test, the results are similar to these reported in Table 8 and are not reported for the sake of brevity; the economic activity (trading activity) model performs best for $h = 1$ month ($h = 2$ months).

7 Conclusions

We examine whether the variance risk premium (VRP) on the S&P 500 can be predicted. To this end, first, we propose a novel approach to measure VRP. We compute VRP as the conditional expectation of the profit and loss obtained from a long position in a variance swap (VS) contract held over an investment horizon which may be shorter than the VS maturity. Our approach is more general than the conventional VRP measure because it disentangles the investment horizon from the VS contract maturity; the conventional VRP measure assumes that the investor keeps her VS position until maturity. We compute the market VRP by employing actual over-the-counter VS quotes written on the S&P 500 as opposed to synthesised VS rates and hence, the computed VRP does not suffer from measurement errors. Finally, we address our research question by developing a predictive setting being navigated by the predictions of financial theory and previous empirical evidence.

We find that economic and trading conditions predict the market VRP; the VRP increases (i.e. becomes more negative) when the economic and trading conditions deteriorate. Our findings hold under both in an in-sample and an out-of-sample statistical setting and they are also economically significant. Trading strategies with variance swaps which exploit the time-variation of VRP with the state of the economy and the trading activity outperform the popular buy-and-hold S&P 500 and short volatility strategies, as well as a naive strategy based on the random walk model.

Our findings open at least two avenues for future research. First, future research should examine whether factors related to ambiguity aversion could also help predicting VRP; Miao et al. (2012) show that ambiguity aversion can explain a sizable portion of the observed VRP. From an empirical perspective, this is challenging because ambiguity aversion is not observable and hence it needs to be measured. Second, a number of studies find that VRP predicts future stock returns. Our results indicate that the reverse is also true; stock conditions drive VRP at least in-sample.

It can well be the case that two-ways effects are present for the economic and trading activity conditions, too. These questions are beyond the scope of the current paper but they deserve to become topics for future research.

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Appendix A: Profit and Loss from a long position in a variance swap

Consider a variance swap (VS) initiated at time t that matures at time $t+T$, $T > 0$. The profit and loss (P&L) of the VS from time t held to $t+T$ is given by:

$$P\&L^{T}_{t\to t+T} = N\left(RV_{t\to t+T} - VS_{t\to t+T}\right) \tag{A.1}$$

where N is the notional value of the VS, $VS_{t\to t+T}$ the VS rate agreed at t for a contract that matures at T (quoted in variance terms),

$$RV_{t\to t+T} = \frac{252}{T}\sum_{i=1}^{T}\ln\left(\frac{S_{t+i}}{S_{t+i-1}}\right)^{2} \tag{A.2}$$

is the realized variance of the underlying return distribution from t to $t+T$ and S_t is the closing price of the S&P 500 index on day t. For an intermediate point in time $t+h$ with $0 < h < T$ the additivity property of the variance dictates that

$$TRV_{t\to t+T} = hRV_{t\to t+h} + (T-h)\,RV_{t+h\to t+T}$$

$$RV_{t\to t+T} = \frac{h}{T}RV_{t\to t+h} + \frac{(T-h)}{T}RV_{t+h\to t+T}$$

$$RV_{t\to t+T} = \lambda RV_{t\to t+h} + (1-\lambda)\,RV_{t+h\to t+T} \tag{A.3}$$

where $\lambda \equiv \frac{h}{T}$ and $(1-\lambda) \equiv \frac{T-h}{T}$. Subtracting $VS_{t\to t+T}$ from both sides of equation (A.3) and re-arranging yields:

$$RV_{t\to t+T} - VS_{t\to t+T} = \lambda\left(RV_{t\to t+h} - VS_{t\to t+T}\right) + (1-\lambda)\left(RV_{t+h\to t+T} - VS_{t\to t+T}\right) \tag{A.4}$$

Substituting equation (A.4) into equation (A.1) we get:

$$P\&L^{T}_{t\to t+T} = N\left[\lambda\left(RV_{t\to t+h} - VS_{t\to t+T}\right) + (1-\lambda)\left(RV_{t+h\to t+T} - VS_{t\to t+T}\right)\right] \tag{A.5}$$

Suppose now that we decide to close our position in this VS at time $t+h$, $0 < h < T$. Recall that the value of the VS at time t is zero, so the price at $t+h$ is also the P&L from t to $t+h$ and the price at time $t+T$ is also the P&L from t to $t+T$. Hence, standing at $t+h$, the P&L from t to $t+h$ is given by the following equation:

$$P\&L^T_{t\to t+h} = e^{-r(T-h)} E^Q_{t+h} \left(P\&L^T_{t\to t+T} \right)$$

$$= e^{-r(T-h)} N E^Q_{t+h} \left[\lambda \left(RV_{t\to t+h} - VS_{t\to t+T} \right) + (1-\lambda) \left(RV_{t+h\to t+T} - VS_{t\to t+T} \right) \right] \tag{A.6}$$

Substituting $VS_{t\to t+T} = E^Q_t \left[RV_{t\to t+T} \right]$ into equation (A.6) yields

$$P\&L^T_{t\to t+h} = e^{-r(T-h)} N \left[\lambda \left(RV_{t\to t+h} - VS_{t\to t+T} \right) + (1-\lambda) \left(VS_{t+h\to t+T} - VS_{t\to t+T} \right) \right] \tag{A.7}$$

Re-arranging equation (A.7) yields

$$P\&L^T_{t\to t+h} = e^{-r(T-h)} N \left[\lambda RV_{t\to t+h} + (1-\lambda) VS_{t+h\to t+T} - VS_{t\to t+T} \right] \tag{A.8}$$

Appendix B: Realized variance forecasts based on a GARCH(1,1)

We construct forecasts for the realized variance under the P-probability measure by using a GARCH (1,1) model that captures the mean-reverting behaviour of the realized variance which has been documented empirically. The GARCH(1,1) specification is defined as follows:

$$r_{t+1} = \mu + \varepsilon_{t+1} \tag{B.1}$$

$$\sigma^2_{t\to t+1} = c + \alpha \varepsilon^2_t + \beta \sigma^2_{t-1\to t} \tag{B.2}$$

where μ is the constant conditional expectation of the returns, $\varepsilon_{t+1} = \sigma_{t+1} z_{t+1}$ is the error term with $z_{t+1}\tilde{\ } N(0,1)$, $\sigma^2_{t\to t+1}$ is the conditional variance between t and

$t+1$ given the information at time t, and c, a and β are constant coefficients. Based on equation (B.2), standing at $t+l-1$, the l-step ahead forecast of the variance between $t+l-1$ and $t+l$ is:

$$E_t\left(\sigma^2_{t+l-1\to t+l}\right) = c\sum_{j=0}^{l-1}(\alpha+\beta)^j + (\alpha+\beta)^l\,\sigma^2_{t-1\to t} \tag{B.3}$$

We construct the conditional expectation of the realized variance between t and $t+l$ as follows. Standing at time t, first we estimate equations (B.1) and (B.2) recursively using a rolling window of 1,000 S&P 500 daily returns. Then, we construct the forecast of the realized variance between t and $t+l$ by averaging the daily one-step ahead obtained forecasted variances.

Appendix C: Construction of model-free risk-neutral skewness

We construct the risk-neutral skewness by implementing the Bakshi et al. (2003) model-free methodology. Let E_t^Q denote the conditional expected value operator under the risk-neutral measure formed at time t, r the risk-free rate, $C(t,\tau;K)$ $[P(t,\tau;K)]$ the price of a call [put] option with time to expiration τ and strike price K and $R(t,\tau)=\ln(S_{t+\tau}/S_t)$ be the continuously compounded rate of return at time t over a time period τ. Let also $V(\bullet)$, $W(\bullet)$ and $X(\bullet)$

$$V(t,\tau) \equiv E_t^Q\left[e^{-r\tau}R(t,\tau)^2\right] \tag{C.1}$$

$$W(t,\tau) \equiv E_t^Q\left[e^{-r\tau}R(t,\tau)^3\right] \tag{C.2}$$

$$W(t,\tau) \equiv E_t^Q\left[e^{-r\tau}R(t,\tau)^4\right] \tag{C.3}$$

denote the fair values of three respective contracts with corresponding payoff functions $H[S]$

$$H[S] = \begin{cases} R(t,\tau)^2 \\ R(t,\tau)^3 \\ R(t,\tau)^4 \end{cases} \tag{C.4}$$

Let $\mu(t,\tau) \equiv E_t^Q \left\{\ln(S_{t+\tau}/S_t)\right\} \approx e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau)$ be the mean of the log-return over period τ. The risk-neutral skewness (SKEW) extracted at time t with horizon τ can be expressed in terms of the fair values of the three artificial contracts, i.e.

$$\begin{aligned} SKEW(t,\tau) &= \frac{E_t^Q \left\{ \left[R(t,\tau) - E_t^Q R(t,\tau)\right]^3 \right\}}{E_t^Q \left\{ \left[R(t,\tau) - E_t^Q R(t,\tau)\right]^2 \right\}^{3/2}} \\ &= \frac{e^{r\tau}W(t,\tau) - 3\mu(t,\tau)e^{r\tau}V(t,\tau) + 2\mu(t,\tau)^3}{\left[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2\right]^{3/2}} \end{aligned} \tag{C.5}$$

Bakshi et al. (2003) show that the arbitrage-free prices of $V(t,\tau)$,$W(t,\tau)$ and $X(t,\tau)$ are given by

$$\begin{aligned} V(t,\tau) &= \int_{S_t}^{\infty} \frac{2\left(1 - \ln(K/S_t)\right)}{K^2} C(t,\tau;K)dK + \\ &+ \int_{0}^{S_t} \frac{2\left(1 + \ln(S_t/K)\right)}{K^2} P(t,\tau;K)dK \end{aligned} \tag{C.6}$$

$$\begin{aligned} W(t,\tau) &= \int_{S_t}^{\infty} \frac{6\ln(K/S_t) - 3\ln(K/S_t)}{K^2} C(t,\tau;K)dK + \\ &+ \int_{0}^{S_t} \frac{6\ln(S_t/K) + 3\ln(S_t/K)}{K^2} P(t,\tau;K)dK \end{aligned} \tag{C.7}$$

$$
\begin{aligned}
X(t,\tau) &= \int\limits_{S_t}^{\infty} \frac{12\left[\ln(K/S_t)\right]^2 - 4\left[\ln(K/S_t)\right]^3}{K^2} C(t,\tau;K)dK + \\
&\quad + \int\limits_{0}^{S_t} \frac{12\left[\ln(S_t/K)\right]^2 + 4\left[\ln(S_t/K)\right]^3}{K^2} \qquad \text{(C.8)}
\end{aligned}
$$

Thus, the price of each contract can be computed as a linear combination of out-of-the-money call and put options. Based on these prices, the risk-neutral skewness is computed in a model-free manner.

Appendix D: Choosing the decay factor in the exponentially weighted average model

We construct forecasts for the realized variance under the P-probability measure by using the exponentially weighted moving average (EWMA) model. Standing at time t, the EWMA model is defined as follows:

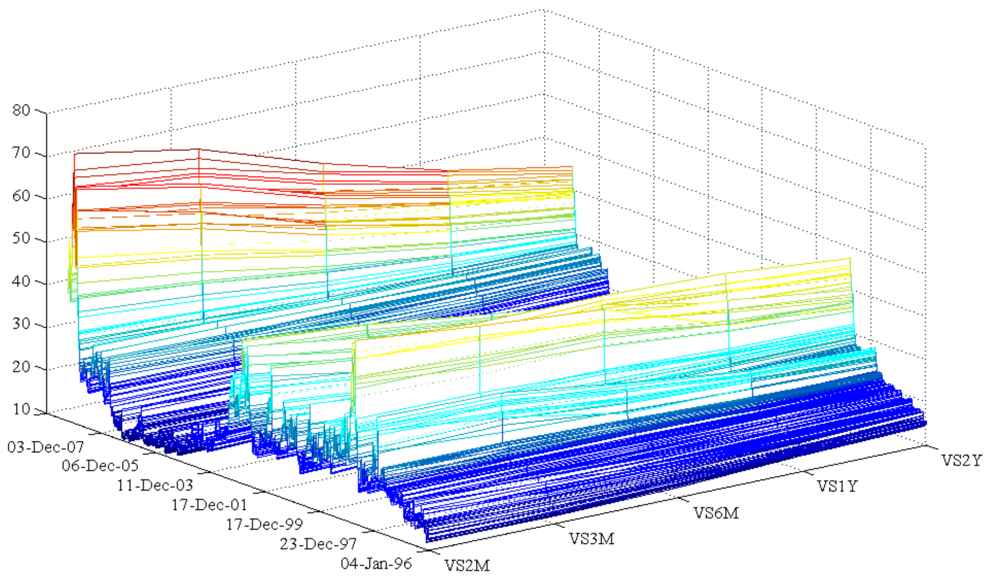
$$
\begin{aligned}
\sigma_{t+1}^2 &= \lambda\sigma_t^2 + (1-\lambda)r_t^2 \qquad \text{(D.1)} \\
&= (1-\lambda)\left(r_t^2 + \lambda r_{t-1}^2 + \lambda^2 r_{t-2}^2 + ...\right) \qquad \text{(D.2)}
\end{aligned}
$$

where λ is the decay factor. We choose λ by setting half life (i.e. the time it takes for the weight to become half of the weight that is used for the current observation, HL), equal to the forecasting horizon, h :

$$
\lambda = \sqrt[HL-1]{\frac{1}{2}} \qquad \text{(D.3)}
$$

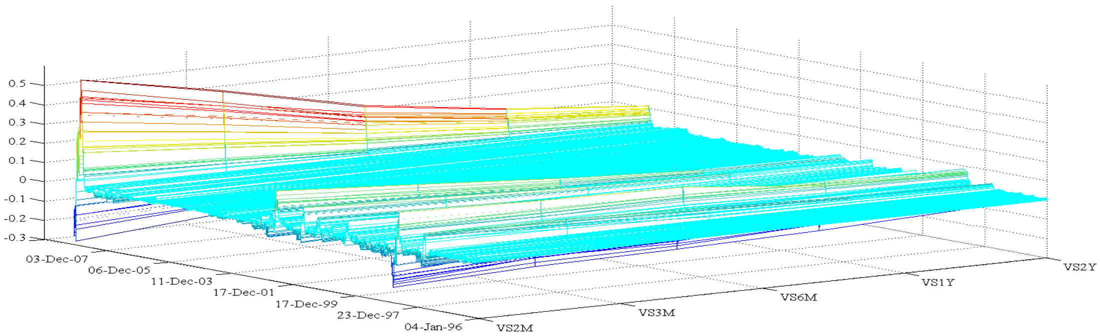
This yields $\lambda = 0.983$ for $h = 2$ months, $\lambda = 0.989$ for $h = 3$ months, $\lambda = 0.994$ for $h = 6$ months, $\lambda = 0.997$ for $h = 1$ year and $\lambda = 0.999$ for $h = 2$ years.

Figure 1: Evolution of variance swap rates

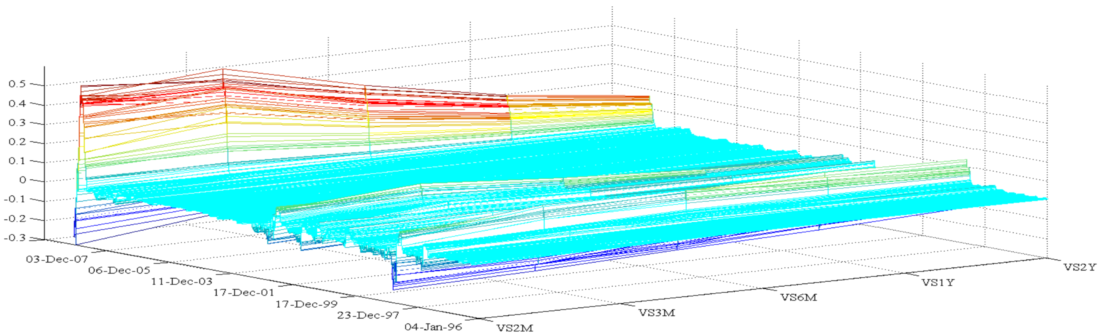


Time series of the S&P 500 variance swap (VS) rates in volatility percentage points with times-to-maturity equal to 2 months (VS2M), 3 months (VS3M), 6 months (VS6M), 1 year (VS1Y) and 2 years (VS2Y). The sample spans January 4, 1996 to February 13, 2009.

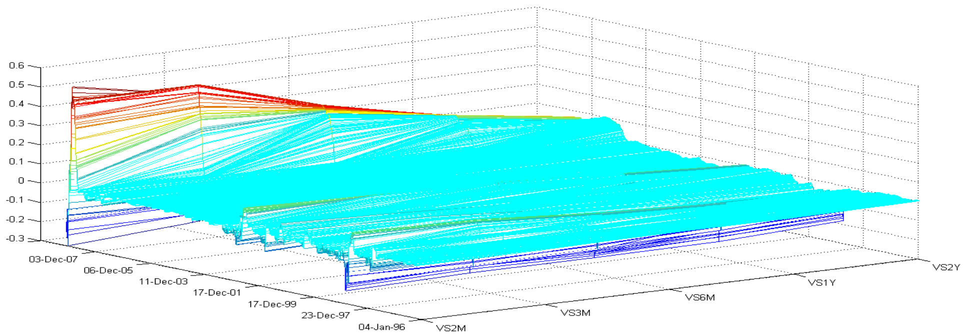
Figure 2: Evolution of S&P 500 variance swap profit and losses



Panel A: P&Ls from holding a VS over one month



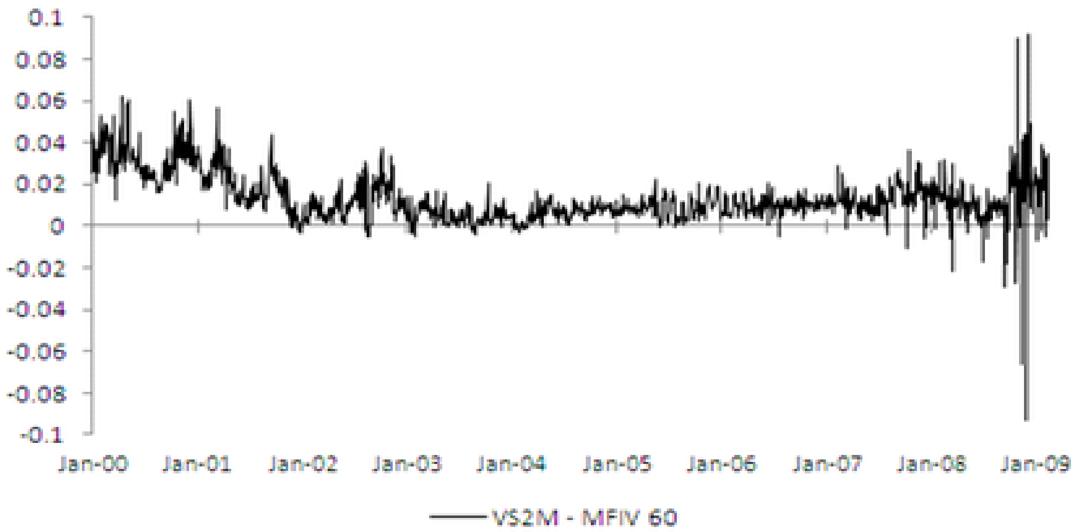
Panel B: P&Ls from holding a VS over two months



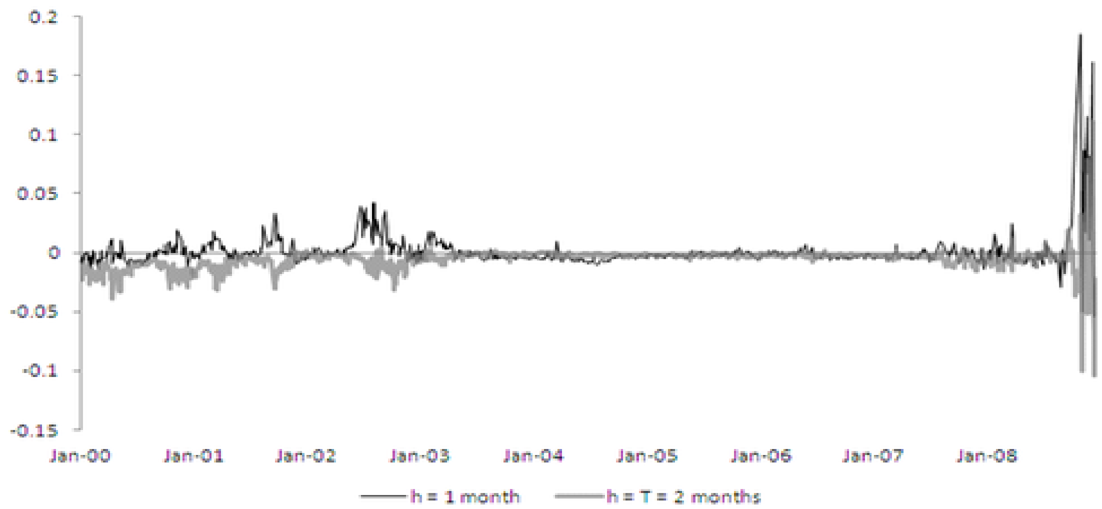
Panel C: P&Ls from holding a VS over T months

Time series of the profit and losses (P&L) obtained from holding over h months a T-maturity variance swap (VS) contract ($T = 2$ months, 3 months, 6 months, 1 year and 2 years). Panels A, B and C show the P&Ls for $h = 1$ month, 2 months and T months, respectively. The P&Ls are recorded at time t, i.e. at the time where a position is opened, and span January 4, 1996 to December 12, 2008.

Figure 3: Bias in the synthesized variance swap rates



Panel A: Difference between the quoted and synthesized VS rate with two months time-to-maturity



Panel B: Difference between the P&Ls based on the quoted and the synthesized VS rates with two months time-to-maturity

Panel A shows the difference between quoted and synthesized two-months maturity VS rates. Panel B shows the difference between VS P&Ls constructed from quoted and synthesized two-months maturity VS rates. The VS P&Ls are computed for two alternative investment horizons: $h = 1$ month (black line) and $h = T = 2$ months (grey line). The figures refer to the period January 3, 2000 to February 13, 2009.

Table 1: Summary statistics for the variance swap rates

	VS2M	**VS3M**	**VS6M**	**VS1Y**	**VS2Y**
# Obs.	3302	3302	3302	3302	3302
Mean	21.7	21.79	22.19	22.77	23.26
Maximum	72.96	67.87	58.39	50.6	47.24
Minimum	10.34	10.92	11.94	13.11	14.01
Std. Dev.	8.32	7.92	7.27	6.67	6.23
Skewness	1.74	1.62	1.31	0.97	0.73
Kurtosis	8.04	7.44	5.8	4.32	3.28

Entries report summary statistics for the variance swap (VS) rates across different contract maturities. VS rates span January 4, 1996 to February 13, 2009.

Table 2: Summary statistics for variance swaps P&Ls

	VS2M	VS3M	VS6M	VS1Y	VS2Y
Panel A: Investment horizon of $h = 1$ month					
Mean	**-0.0042***	**-0.0024***	**-0.0011****	-0.0004	0.0001
Maximum	0.56	0.43	0.27	0.17	0.11
Minimum	-0.28	-0.22	-0.18	-0.16	-0.13
Std. Dev.	0.05	0.04	0.03	0.02	0.02
Skewness	4.85	4.18	2.91	1.95	1.06
Kurtosis	47.50	41.15	29.48	22.71	16.80
Panel B: Investment horizon of $h = 2$ months					
Mean	**-0.0100***	**-0.0054***	**-0.0024***	-0.0008	0.0002
Median	-0.013	-0.010	-0.006	-0.003	-0.002
Maximum	0.53	0.54	0.38	0.25	0.16
Minimum	-0.30	-0.22	-0.18	-0.16	-0.13
Std. Dev.	0.06	0.06	0.04	0.03	0.02
Skewness	4.83	4.71	4.08	3.41	2.58
Kurtosis	40.62	37.57	31.96	25.31	18.49
Panel C: Investment horizon of $h = T$ months					
Mean	**-0.010***	**-0.009***	**-0.009***	**-0.010***	**-0.011***
Median	-0.013	-0.013	-0.013	-0.014	-0.013
Maximum	0.53	0.47	0.28	0.16	0.10
Minimum	-0.30	-0.26	-0.22	-0.21	-0.18
Std. Dev.	0.06	0.06	0.05	0.05	0.04
Skewness	4.83	4.48	2.79	1.38	0.55
Kurtosis	40.62	33.85	15.67	7.42	3.97

Entries report summary statistics for the profit and losses (P&L) from investing in VS contracts of different maturities and over different investment horizons h ($h = 1$, 2 and T months in panels A, B and C, respectively). One and two asterisks denote rejection of the null hypothesis of a zero average (unconditional) P&L at the 1% and 5% level, respectively. P&Ls span January 4, 1994 to December 12, 2008.

Table 3: **Test for time variation of variance swap P&Ls**

	VS2M	**VS3M**	**VS6M**	**VS1Y**	**VS2Y**
Panel A: Investment horizon of h = 1 month					
$\boldsymbol{a}$	**0.019***	**0.015***	**0.008***	**0.004***	**0.002***
$(H_0 : a = 0)$	*(3.667)*	*(4.855)*	*(7.332)*	*(9.182)*	*(10.462)*
$\boldsymbol{b}$	**0.078***	**-0.057***	**-0.162***	**-0.138***	**-0.088***
$(H_0 : b = 1)$	*(-5.222)*	*(-6.943)*	*(-12.432)*	*(-20.199)*	*(-34.462)*
Panel B: Investment horizon of h = 2 months					
$\boldsymbol{a}$	0.002	**0.024***	**0.017***	**0.009***	**0.004***
$(H_0 : a = 0)$	*(0.329)*	*(3.578)*	*(6.246)*	*(8.982)*	*(11.031)*
$\boldsymbol{b}$	**-0.773***	**0.116***	**-0.199***	**-0.204***	**-0.140***
$(H_0 : b = 1)$	*(-13.407)*	*(-5.099)*	*(-9.146)*	*(-15.777)*	*(-26.329)*
Panel C: Investment horizon of h = T months					
$\boldsymbol{a}$	0.002	0.008	**0.015***	**0.022***	**0.038***
$(H_0 : a = 0)$	*(0.329)*	*(1.730)*	*(3.690)*	*(6.859)*	*(10.639)*
$\boldsymbol{b}$	**-0.773***	**-0.670***	**-0.543***	**-0.410***	**0.137***
$(H_0 : b = 1)$	*(-13.407)*	*(-16.907)*	*(-18.988)*	*(-25.122)*	*(-19.866)*

Entries report results from the estimated equation (7). Coefficient estimates and the Newey-West t-statistics (within parentheses) are reported. We test two null hypotheses, namely that the constant equals zero ($H_0 : a = 0$) and that the slope coefficient equals one ($H_0 : b = 1$). One and two asterisks denote rejection of the null hypothesis at a 1% and 5% level, respectively. The sample spans January 4, 1994 to December 12, 2008.

Table 4: List of candidate VRP predictors

Predictor	Construction	Sign
Panel A: Volatility variation		
Volatility of volatility (VoV)	$VoV_t = VS_t^2 - VolS_t^2$ where VS_t is the variance swap rate and $VolS_t$ is the volatility swap rate with 2 months time to maturity VolS is proxied by the S&P 500 at-the-money implied volatility.	$\downarrow$
Correlation (Corr)	Rolling correlation of the daily S&P 500 returns and the daily changes of VIX over the past year.	$\uparrow$
Panel B: Stock market conditions		
Market volatility (VIX)	CBOE VIX index.	$\downarrow$
Return (R)	Returns of the S&P 500 index between t and $t + h$.	$\uparrow$
Market skewness (SKEW)	CBOE SKEW index.	$\downarrow$*
Ex-ante VRP (VRP)	Ex-ante VRP for the S&P 500 with an textith-months horizon [equation (1)]. We use a GARCH(1,1) model to construct the expected realized variance under the P-probability measure and the squared variance swap rate to measure the risk-neutral expectation of realized variance.	$\uparrow$
Panel C: Economic conditions		
Term spread (TS)	Difference between the ten-years US government bond and the one-month LIBOR rate.	$\uparrow$
Credit spread (CS)	Difference between the Moody's AAA and BAA.	$\downarrow$
Panel D: Trading activity		
Underlying asset volume (SVol)	Shortest S&P 500 futures volume.	$\downarrow$
Aggregate futures volume (TVol)	Total S&P 500 futures volume.	$\uparrow$
Demand for hedging tail risk (OI)	Out-of-the-money put open interest.	$\downarrow$
Market sentiment (P/C)	Ratio of S&P 500 put volume over S&P 500 call volume.	$\downarrow$
Funding liquidity (TED)	Difference between the three-months Eurodollar rate and the three-months T-BIll rate.	$\downarrow$

Entries provide a brief description of all VRP predictors considered in equation (6) and their measurement (first and second column, respectively). The third column reports the expected sign between VRP and the corresponding VRP predictor. Following previous literature, we use the following convention: given that the market VRP is on average negative, a $\downarrow$($\uparrow$) signifies that VRP increases (decreases), i.e. it becomes more negative (less negative), as the predictor variable increases.

* Market skewness is negative. In general, we expect a $\uparrow$ relation between VRP and market skewness: VRP increases (i.e. it becomes more negative) when the market skewness increases (i.e. it becomes more negative). However, higher values of the CBOE Skew index signify that market skewness becomes more negative and hence, we expect a $\downarrow$ relation with VRP.

Table 5: Single predictor models: In-sample results

	$h = 1$ month					$h = 2$ months					$h = T$ months				
	VS2M	VS3M	VS6M	VS1Y	VS2Y	VS2M	VS3M	VS6M	VS1Y	VS2Y	VS2M	VS3M	VS6M	VS1Y	VS2Y
Panel A: Volatility variation															
Corr	0.103*	0.078*	0.059*	0.037**	0.024	0.127*	0.135*	0.108*	0.071*	0.045*	0.127*	0.144*	0.183*	0.213*	0.238*
	(5.372)	(4.517)	(3.531)	(2.573)	(1.862)	(6.437)	(6.736)	(5.532)	(4.176)	(3.040)	(6.437)	(7.601)	(9.190)	(11.469)	(14.813)
VoV	-1.171*	-0.907*	-0.679	-0.344	-0.218	-1.749*	-1.710*	-1.443*	-0.975*	-0.713*	-1.749*	-1.802*	-1.967*	-1.778*	-1.634*
	(-3.499)	(-3.028)	(-1.948)	(-1.038)	(-0.743)	(-11.353)	(-11.721)	(-8.031)	(-5.156)	(-4.317)	(-11.353)	(-14.268)	(-12.256)	(-9.108)	(-7.812)
Panel B: Stock market conditions															
VIX	-0.003*	-0.003*	-0.002*	-0.001	-0.001	-0.004*	-0.004*	-0.003*	-0.002*	-0.002*	-0.004*	-0.005*	-0.005*	-0.005*	-0.005*
	(-4.764)	(-4.133)	(-2.651)	(-1.598)	(-1.213)	(-8.323)	(-8.422)	(-5.864)	(-4.279)	(-3.605)	(-8.323)	(-10.906)	(-12.256)	(-11.509)	(-13.577)
R	0.086	0.076	0.051	0.017	0.017	0.213*	0.203*	0.203*	0.152*	0.130*	0.213*	0.200*	0.260*	0.239*	0.052
	(1.219)	(1.256)	(0.879)	(0.339)	(0.395)	(3.089)	(3.018)	(3.375)	(3.257)	(3.571)	(3.089)	(3.085)	(4.790)	(4.753)	(1.418)
Skew	-0.001*	-0.001**	-0.001	0.000	0.000	-0.002*	-0.002*	-0.001*	-0.001	0.000	-0.001*	-0.002*	-0.003*	-0.003*	-0.003*
	(-2.843)	(-2.254)	(-1.302)	(-0.585)	(-0.158)	(-4.030)	(-3.957)	(-2.660)	(-1.729)	(-1.037)	(-4.030)	(-4.966)	(-6.503)	(-8.455)	(-12.235)
VRP	0.521	0.452**	0.371	0.263	0.226	0.844*	0.828*	0.661*	0.436**	0.320**	0.844*	0.886*	0.963*	0.773*	0.441
	(1.942)	(1.993)	(1.651)	(1.313)	(1.360)	(3.694)	(3.895)	(3.117)	(2.300)	(2.023)	(3.694)	(3.957)	(4.250)	(3.080)	(1.689)
Panel C: Economic conditions															
TS	1.357*	0.917	0.503	0.158	-0.022	1.957*	1.904*	1.158	0.542	0.166	1.957*	2.212*	2.576*	2.834*	2.988*
	(2.533)	(1.957)	(1.063)	(0.401)	(-0.065)	(3.380)	(3.156)	(1.913)	(1.083)	(0.403)	(3.380)	(4.031)	(4.646)	(6.363)	(8.415)
CS	-8.261*	-6.868*	-6.245*	-4.984*	-3.739*	-9.214*	-11.006*	-10.279*	-8.544*	-6.521*	-9.214*	-10.77*	-13.506*	-16.024*	-15.820*
	(-4.762)	(-4.279)	(-4.181)	(-3.976)	(-3.274)	(-5.357)	(-5.832)	(-5.542)	(-5.230)	(-4.372)	(-5.357)	(-6.472)	(-8.641)	(-13.518)	(-13.430)
Panel D: Trading activity conditions															
SVol	0.006	0.004	0.002	0.002	0.001	0.009	0.009	0.007	0.004	0.003	0.009	0.010	0.013	0.015*	0.019*
	(1.449)	(1.205)	(1.128)	(0.922)	(0.682)	(1.534)	(1.334)	(1.216)	(1.041)	(0.800)	(1.534)	(1.817)	(1.928)	(2.676)	(3.045)
TVol	0.151*	0.118**	0.084	0.055	0.046	0.330*	0.398*	0.408*	0.336**	0.285**	0.330*	0.332*	0.231*	0.214*	0.190*
	(2.646)	(2.057)	(1.650)	(1.470)	(1.272)	(3.338)	(2.834)	(2.580)	(2.451)	(2.222)	(3.338)	(2.760)	(3.013)	(3.233)	(2.332)
OI	-0.012	-0.011	-0.013**	-0.012*	-0.009**	-0.013	-0.013	-0.013	-0.012*	-0.009*	-0.013	-0.013	-0.014	-0.013	-0.01
	(-1.467)	(-1.576)	(-1.967)	(-2.631)	(-2.501)	(-1.440)	(-1.550)	(-1.934)	(-2.642)	(-2.706)	(-1.440)	(-1.448)	(-1.536)	(-1.634)	(-1.383)
P/C	0.158	0.271	0.505**	0.492	0.425	-0.073	0.027	0.290	0.360	0.353	-0.073	-0.075	-0.073	-0.041	-0.098
	(0.482)	(1.001)	(2.009)	(1.813)	(1.872)	(-0.252)	(0.140)	(1.560)	(1.725)	(1.847)	(-0.252)	(-0.369)	(-0.435)	(-0.235)	(-0.583)
TED	-0.046*	-0.038**	-0.038*	-0.031**	-0.024**	-0.055*	-0.056*	-0.054*	-0.044*	-0.033*	-0.055*	-0.060*	-0.068*	-0.087*	-0.089*
	(-2.687)	(-2.504)	(-2.650)	(-2.504)	(-2.344)	(-3.201)	(-3.412)	(-3.671)	(-3.634)	(-3.222)	(-3.201)	(-3.688)	(-3.946)	(-6.702)	(-8.459)

Entries report results from the estimated in-sample single predictor models for any given considered predictor and investment horizon across the various VS maturities. Coefficient estimates and the Newey-West t-statistics of each one of the predictor variables are reported. One and two asterisks denote rejection of the null hypothesis of a zero coefficient at the 1% and 5% level, respectively. We measure all VRP predictors over the common period January 4, 1996 to December 31, 1999.

Table 6: Multiple predictor models: In-sample results

	$h=$ 1 month					$h=$ 2 months					$h=T$ months				
	VS2M	VS3M	VS6M	VS1Y	VS2Y	VS2M	VS3M	VS6M	VS1Y	VS2Y	VS2M	VS3M	VS6M	VS1Y	VS2Y
Panel A: Volatility variation															
C	0.019	0.015	0.014	0.013	0.009	0.000	0.011	0.012	0.008	0.002	0.000	0.011	0.036*	0.069*	0.100
	(1.505)	(1.330)	(1.218)	(1.203)	(0.899)	(0.017)	(1.192)	(1.242)	(0.810)	(0.225)	(0.017)	(1.328)	(4.375)	(7.755)	11.512
Corr	0.026	0.018	0.014	0.014	0.008	0.002	0.012	0.004	-0.002	-0.011	0.002	0.019	0.056*	0.111*	0.157
	(1.238)	(0.922)	(0.646)	(0.706)	(0.449)	(0.123)	(0.763)	(0.240)	(-0.129)	(-0.694)	(0.123)	(1.383)	(4.103)	(7.907)	11.713
VoV	-1.089*	-0.852**	-0.636	-0.3	-0.192	-1.744*	-1.671*	-1.430*	-0.983*	-0.747*	-1.744*	-1.743*	-1.790*	-1.426*	-1.138
	(-2.877)	(-2.511)	(-1.595)	(-0.790)	(-0.570)	(-9.846)	(-9.984)	(-6.925)	(-4.530)	(-3.953)	(-9.846)	(-12.073)	(-10.774)	(-8.142)	-7.168
Adj. R^2	0.291	0.231	0.154	0.059	0.032	0.608	0.498	0.398	0.254	0.2	0.608	0.665	0.777	0.83	0.845
Panel B: Stock market conditions															
C	0.019	0.012	-0.004	-0.010	-0.016	0.050	0.063	0.026	-0.002	-0.023	0.050	0.069	0.053	0.112*	0.184*
	(0.380)	(0.259)	(-0.081)	(-0.253)	(-0.476)	(1.184)	(1.280)	(0.488)	(-0.036)	(-0.605)	(1.184)	(1.834)	(1.675)	(4.535)	(8.940)
VIX	-0.004*	-0.003*	-0.002*	-0.001	-0.001	-0.004*	-0.004*	-0.003*	-0.002*	-0.001*	-0.004*	-0.004*	-0.004*	-0.004*	-0.004*
	(-5.456)	(-4.769)	(-3.049)	(-1.921)	(-1.378)	(-10.257)	(-8.542)	(-5.997)	(-4.828)	(-4.280)	(-10.257)	(-10.514)	(-14.653)	(-10.903)	(-12.612)
R	-0.081	-0.056	-0.048	-0.042	-0.024	0.057**	0.047	0.087*	0.078*	0.081*	0.057**	0.061**	0.128*	0.118*	0.092*
	(-1.165)	(-0.917)	(-0.810)	(-0.850)	(-0.583)	(2.372)	(1.912)	(3.182)	(2.893)	(3.385)	(2.372)	(2.397)	(5.515)	(6.666)	(7.041)
Skew	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	-0.001
	(0.819)	(0.823)	(0.896)	(0.862)	(0.991)	(0.299)	(0.092)	(0.515)	(0.802)	(1.237)	(0.299)	(-0.078)	(0.117)	(-3.095)	(-6.858)
VRP	0.052	0.088	0.109	0.122	0.137	0.158	0.138	0.123	0.074	0.064	0.158	0.134	0.178**	0.400*	0.337*
	(0.282)	(0.516)	(0.590)	(0.661)	(0.859)	(1.163)	(1.150)	(0.834)	(0.502)	(0.514)	(1.163)	(1.335)	(2.431)	(4.611)	(5.134)
Adj. R^2	0.299	0.245	0.157	0.081	0.059	0.541	0.463	0.339	0.219	0.183	0.541	0.621	0.762	0.784	0.794
Panel C: Economic conditions															
C	0.029**	0.030**	0.036*	0.033*	0.028*	0.019	0.038*	0.052*	0.053*	0.046*	0.019	0.026	0.040*	0.054*	0.052*
	(1.962)	(2.234)	(2.964)	(3.405)	(3.335)	(1.226)	(2.078)	(2.953)	(3.483)	(3.596)	(1.226)	(1.670)	(2.851)	(5.372)	(5.479)
CS	-7.120*	-6.157*	-5.971*	-5.034*	-3.937*	-7.438*	-9.378*	-9.459*	-8.333*	-6.641*	-7.438*	-8.782*	-11.245*	-13.575*	-13.182*
	(-3.873)	(-3.670)	(-3.928)	(-3.992)	(-3.513)	(-3.870)	(-4.308)	(-4.474)	(-4.548)	(-4.167)	(-3.870)	(-4.675)	(-6.538)	(-9.842)	(-9.346)
TS	1.053	0.654	0.248	-0.057	-0.19	1.639*	1.503**	0.753	0.186	-0.118	1.639*	1.836*	2.095*	2.253*	2.425*
	(1.942)	(1.385)	(0.522)	(-0.144)	(-0.572)	(2.720)	(2.362)	(1.182)	(0.352)	(-0.271)	(2.720)	(3.205)	(3.677)	(5.084)	(6.727)
Adj. R^2	0.132	0.106	0.089	0.079	0.066	0.197	0.199	0.151	0.130	0.109	0.197	0.264	0.359	0.547	0.579
Panel D: Trading activity conditions															
C	0.01	0.011	0.016**	0.015**	0.013*	0.005	0.01	0.019*	0.020*	0.018*	0.005	0.006	0.008	0.018*	0.020*
	(1.127)	(1.431)	(2.266)	(2.492)	(2.578)	(0.617)	(1.209)	(2.642)	(3.335)	(3.428)	(0.617)	(0.755)	(0.971)	(2.664)	(3.581)
TVol	0.000	0.000	0.000	0.000	0.000	0.000**	0.000**	0.000**	0.000**	0.000	0.000**	0.000**	0.000**	0.000*	0.000*
	(1.953)	(1.467)	(1.042)	(0.847)	(0.751)	(2.452)	(2.229)	(2.064)	(1.968)	(1.840)	(2.452)	(2.049)	(2.038)	(4.141)	(2.811)
TED	-0.046*	-0.038**	-0.038*	-0.031**	-0.024**	-0.055*	-0.056*	-0.054*	-0.044*	-0.033*	-0.055*	-0.060*	-0.068*	-0.087*	-0.088*
	(-2.683)	(-2.500)	(-2.649)	(-2.504)	(-2.344)	(-3.193)	(-3.403)	(-3.666)	(-3.631)	(-3.218)	(-3.193)	(-3.679)	(-3.939)	(-6.694)	(-8.447)
Adj. R^2	0.089	0.080	0.099	0.094	0.081	0.126	0.113	0.118	0.110	0.092	0.126	0.152	0.177	0.335	0.355

Entries report results from the in-sample estimated multiple predictor models for any given considered variable and investment horizon across the various VS maturities. Coefficient estimates and the Newey-West t-statistics of each one of the predictor variables are reported. One and two asterisks denote rejection of the null hypothesis of a zero coefficient at the 1% and 5% level, respectively. We measure all VRP predictors over the common period January 4, 1996 to December 31, 1999.

Table 7: Out-of-sample evaluation: R^2 and MCP

	$h = 1$ month					$h = 2$ months					$h = T$ months				
	VS2M	**VS3M**	**VS6M**	**VS1Y**	**VS2Y**	**VS2M**	**VS3M**	**VS6M**	**VS1Y**	**VS2Y**	**VS2M**	**VS3M**	**VS6M**	**VS1Y**	**VS2Y**
Panel A: Out of sample R^2															
Volatility-variation	0.154	0.101	0.056	0.086	0.185	0.430	0.439	0.434	0.433	0.432	0.430	0.448	0.039	-0.435	-0.176
Stock market conditions	-0.666	-0.640	-0.517	-0.390	-0.179	-0.942	-1.068	-0.996	-0.839	-0.615	-0.942	-0.083	-0.002	-0.974	-0.598
Economic conditions	-0.096	-0.134	-0.119	-0.049	0.091	0.069	0.060	0.080	0.110	0.164	0.069	0.149	-0.030	-0.257	0.371
Trading activity	0.139	0.120	0.138	0.178	0.248	0.371	0.384	0.394	0.408	0.425	0.371	0.405	0.282	0.279	-1.362
Panel B: Mean Correct Prediction															
Volatility-variation	**65.3%***	**60.5%***	**59.2%***	**58.4%***	51.1%	**70.0%***	**62.0%***	**61.4%***	**61.6%***	**58.1%***	**70.0%***	**67.6%***	**62.8%***	**53.0%***	**39.8%***
	(13.279)	*(9.076)*	*(7.967)*	*(7.275)*	*(0.947)*	*(17.260)*	*(10.356)*	*(9.800)*	*(9.986)*	*(7.020)*	*(17.260)*	*(15.137)*	*(10.889)*	*(2.496)*	*(-8.034)*
Stock market conditions	**59.3%***	**55.4%***	**54.3%***	**54.1%***	50.2%	**67.9%***	**59.9%***	**60.2%***	**63.0%***	**58.7%***	**67.9%***	**64.5%***	**60.0%***	43.2%	34.1%
	(8.806)	*(5.071)*	*(4.053)*	*(3.841)*	*(0.233)*	*(16.800)*	*(9.253)*	*(9.551)*	*(12.195)*	*(8.187)*	*(16.800)*	*(13.560)*	*(9.217)*	*(-6.040)*	*(-13.258)*
Economic conditions	**68.4%***	**66.0%***	**63.4%***	**60.3%***	**54.9%***	**76.6%***	**69.4%***	**68.2%***	**65.6%***	**59.1%***	**76.6%***	**73.2%***	**71.6%***	**60.6%***	**66.1%***
	(17.342)	*(15.047)*	*(12.624)*	*(9.649)*	*(4.633)*	*(24.920)*	*(18.215)*	*(17.019)*	*(14.585)*	*(8.563)*	*(24.920)*	*(21.629)*	*(19.881)*	*(9.429)*	*(13.372)*
Trading activity	**61.1%***	**58.4%***	**56.4%***	**51.6%**	**50.0%**	**72.8%***	**62.7%***	**61.4%***	**58.5%***	**55.3%***	**72.8%***	**67.8%***	**69.4%***	**74.1%***	**71.8%***
	(10.466)	*(7.913)*	*(6.041)*	*(1.532)*	*(0.000)*	*(21.309)*	*(11.905)*	*(10.665)*	*(7.972)*	*(4.980)*	*(21.309)*	*(16.537)*	*(17.764)*	*(21.440)*	*(18.176)*

Entries report the out-of-sample R^2 and Mean Correct Prediction (MCP) for any given model specification across the various VS contracts and investment horizons (panels A and B, respectively). One and two asterisks denote rejection of the null hypothesis that MCP is less than 50% at the 1% and 5% level, respectively, based on the ratio test. We construct the forecasts using the multiple predictors models. Each model is estimated using a rolling window of 1,009 daily observations, At each time step, VRP's predictors are measured over a common in-sample period. The first in-sample period for all VRP predictors spans January 4, 1996 to December 31, 1999, whereas the last one spans December 13, 2004 to December 12, 2008.

Table 8: Sharpe ratios after transaction costs

	VS2M	VS3M	VS6M	VS1Y	VS2Y
Panel A: Investment horizon of $h = 1$ month					
Volatility variation model	0.11	0.10	0.12	0.10	0.23
Stock market conditions model	-0.21	-0.20	-0.24	-0.25	-0.26
Economic conditions model	0.43	0.28	0.27	0.34	0.41
Trading activity model	-0.28	-0.35	0.02	0.07	0.09
Buy-and-hold strategy (Long S&P 500)	-0.09	-0.09	-0.09	-0.09	-0.09
Short VS	0.07	0.03	0.02	-0.01	-0.04
Random walk model	0.13	0.14	0.18	0.17	0.15
Panel B: Investment horizon of $h = 2$ months					
Volatility variation model	-0.12	-0.02	-0.16	-0.33	-0.74
Stock market conditions model	-0.16	-0.25	-0.29	-0.35	-0.30
Economic conditions model	-0.16	-0.26	-0.31	-0.25	0.27
Trading activity model	-0.46	-0.27	0.36	0.44	0.56
Buy-and-hold strategy (Long S&P 500)	-0.12	-0.12	-0.12	-0.12	-0.12
Short VS	0.12	0.02	0.00	-0.03	-0.06
Random walk model	0.13	0.04	0.17	0.21	0.12
Panel C: Investment horizon of $h = T$ months					
Volatility variation model	-0.12	-0.19	-0.21	-0.52	-0.42
Stock market conditions model	-0.16	-0.21	-0.29	-0.47	-0.55
Economic conditions model	-0.16	-0.29	-0.37	-0.29	-0.19
Trading activity model	-0.46	-0.46	-0.10	1.12	0.2
Buy-and-hold strategy (Long S&P 500)	-0.12	-0.13	-0.13	-0.11	0.04
Short VS	0.12	0.07	0.01	-0.01	-0.17
Random walk model	0.13	0.09	-0.34	0.84	-0.14

Entries report the Sharpe ratio after transaction costs for any given model specification and benchmark strategies across the various variance swap contract maturities and investment horizons. The employed trading strategy is the following: Go long (short) a variance swap contract when the forecasted profit and loss (P&L) is greater (less) than the filter value (minus the filter value) and keep this contract for an h investment horizon ($h = 1$, 2, and T months in panels A, B and C respectively). If this trading condition is not met, the investor stays out of the market. The filter equals one standard deviation of the P&Ls used for the in-sample estimation of each model specification at each time step. The transaction cost equals 0.5 volatility points per transaction.